HAMPSTEAD SCHOOL
Learning together Achieving together

Y11 – Y12
Summer Bridging Tasks
2025

Physics

Name: ____

- You should spend some time during the summer holidays working on the activities in this booklet.
- You will be required to hand this work in during your first lesson at the start of Year 12 and the content may be used to form the basis of your first assessments.

- You should try your best and show commitment to your studies.
- We are really looking forward to you coming to Hampstead School Sixth Form and studying Physics

Transition Pack for A Level Physics

Get ready for A-level!

A guide to help you get ready for A-level Physics, including everything from topic guides to days out and online learning courses.

Commissioned by The PiXL Club Ltd. February 2016

Please note: these resources are non-board specific. Please direct your students to the specifics of where this knowledge and skills most apply.

So you are considering A Level Physics?

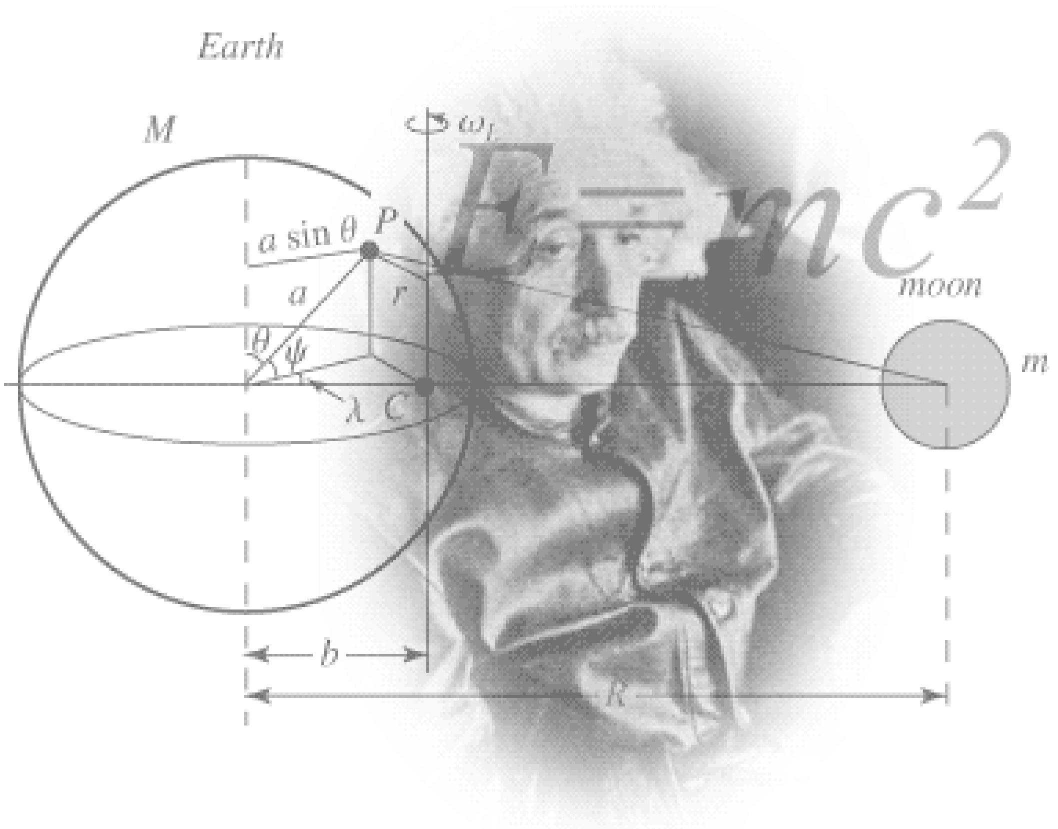


Figure 1 http://scienceworld.wolfram.com/physics/images/main-physics.gif

This pack contains a programme of activities and resources to prepare you to start an A level in Physics in September. It is aimed to be used after you complete your GCSE, throughout the remainder of the Summer term and over the Summer Holidays to ensure you are ready to start your course in September.

Pre-Knowledge Topics

Below are ten topics that are essential foundations for you study of A-Level Physics. Each topics has example questions and links where you can find our more information as you prepare for next year.

Symbols and Prefixes

Prefix	Symbol	Power of ten
Nano	n	x 10^{-9}
Micro	μ	x 10^{-6}
Milli	m	x 10^{-3}
Centi	c	x 10^{-2}
Kilo	k	x 10^{3}
Mega	M	x 10^{6}
Giga	G	x 10^{9}

At A level, unlike GCSE, you need to remember all symbols, units and prefixes. Below is a list of quantities you may have already come across and will be using during your A level course

Quantity	Symbol	Unit
Velocity	v	ms^{-1}
Acceleration	a	ms^{-2}
Time	t	S
Force	F	N
Resistance	R	Ω
Potential difference	V	V
Current	I	A
Energy	E or W	J
Pressure	P	Pa
Momentum	p	$kgms^{-1}$
Power	P	W
Density	ρ	kgm^{-3}
Charge	Q	C

Solve the following:

1. How many metres in 2.4 km?
2. How many joules in 8.1 MJ?
3. Convert 326 GW into W.
4. Convert 54600 mm into m.
5. How many grams in 240 kg?
6. Convert 0.18 nm into m.
7. Convert 632 nm into m. Express in standard form.
8. Convert 1002 mV into V. Express in standard form.
9. How many eV in 0.511 MeV? Express in standard form.
10. How many m in 11 km? Express in standard form.

Standard Form

At A level quantity will be written in standard form, and it is expected that your answers will be too.

This means answers should be written as ….x 10^y. E.g. for an answer of 1200kg we would write 1.2 x 10^3kg. For more information visit: www.bbc.co.uk/education/guides/zc2hsbk/revision

1. Write 2530 in standard form.
2. Write 280 in standard form.
3. Write 0.77 in standard form.
4. Write 0.0091 in standard form.
5. Write 1 872 000 in standard form.
6. Write 12.2 in standard form.
7. Write 2.4 x 10^2 as a normal number.
8. Write 3.505 x 10^1 as a normal number.
9. Write 8.31 x 10^6 as a normal number.
10. Write 6.002 x 10^2 as a normal number.
11. Write 1.5 x 10^{-4} as a normal number.
12. Write 4.3 x 10^3 as a normal number.

Rearranging formulae

This is something you will have done at GCSE and it is crucial you master it for success at A level. For a recap of GCSE watch the following links:

www.khanacademy.org/math/algebra/one-variable-linear-equations/old-school-equations/v/solving-for-a-variable

www.youtube.com/watch?v=_WWgc3ABSj4

Rearrange the following:

1. E=m x g x h to find h
2. Q= I x t to find I
3. E = ½ m v^2 to find m
4. E = ½ m v^2 to find v
5. v = u + at to find u
6. v = u + at to find a
7. $v^2 = u^2 + 2as$ to find s
8. $v^2 = u^2 + 2as$ to find u

Significant figures

At A level you will be expected to use an appropriate number of significant figures in your answers. The number of significant figures you should use is the same as the number of significant figures in the data you are given. You can never be more precise than the data you are given so if that is given to 3 significant your answer should be too. E.g. Distance = 8.24m, time = 1.23s therefore speed = 6.75m/s

The website below summarises the rules and how to round correctly.

http://www.purplemath.com/modules/rounding2.htm

Give the following to 3 significant figures:

1. 3.4527
2. 40.691
3. 0.838991
4. 1.0247
5. 59.972

Calculate the following to a suitable number of significant figures:

6. 63.2/78.1
7. 39+78+120
8. (3.4+3.7+3.2)/3
9. 0.0256 x 0.129
10. 592.3/0.1772

Recording Data

Whilst carrying out a practical activity you need to write all your raw results into a table. Don't wait until the end, discard anomalies and then write it up in neat.

Tables should have column heading and units in this format quantity/unit e.g. length /mm

All results in a column should have the same precision and if you have repeated the experiment you should calculate a mean to the same precision as the data.

Below are link to practical handbooks so you can familiarise yourself with expectations.

http://filestore.aqa.org.uk/resources/physics/AQA-7407-7408-PHBK.PDF

http://www.ocr.org.uk/Images/295483-practical-skills-handbook.pdf

http://www.ocr.org.uk/Images/295483-practical-skills-handbook.pdf

Below is a table of results from an experiment where a ball was rolled down a ramp of different lengths. A ruler and stop clock were used.

1) Identify the errors the student has made.

	Time			
Length/cm	**Trial 1**	**Trial 2**	**Trial 3**	**Mean**
10	1.45	1.48	1.46	1.463
22	2.78	2.72	2.74	2.747
30	4.05	4.01	4.03	4.03
41	5.46	5.47	5.46	5.463
51	7.02	6.96	6.98	6.98
65	8.24	9.68	8.24	8.72
70	9.01	9.02	9.0	9.01

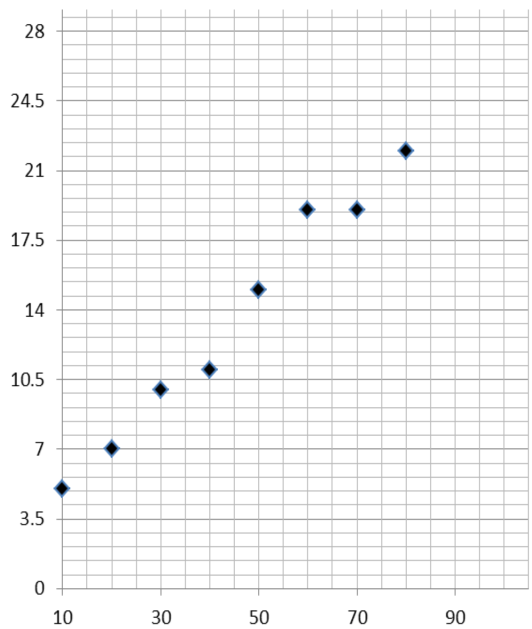
Graphs

After a practical activity the next step is to draw a graph that will be useful to you. Drawing a graph is a skill you should be familiar with already but you need to be extremely vigilant at A level. Before you draw your graph to need to identify a suitable scale to draw taking the following into consideration:

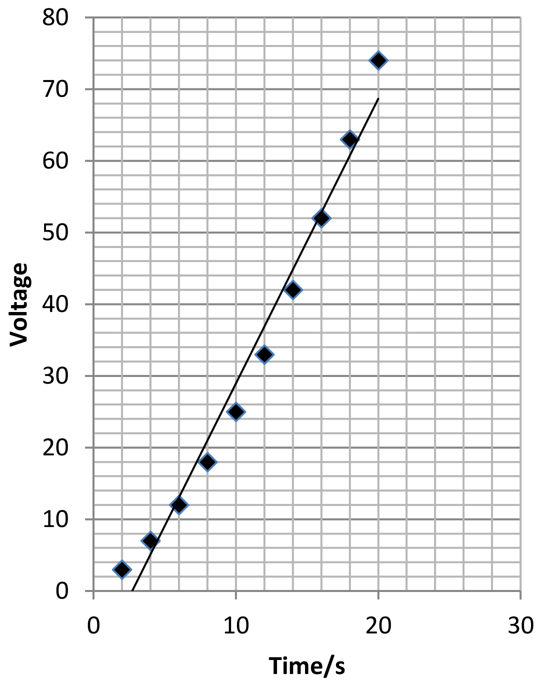
- the maximum and minimum values of each variable
- whether 0.0 should be included as a data point; graphs don't need to show the origin, a false origin can be used if your data doesn't start near zero.
- the plots should cover at least half of the grid supplied for the graph.
- the axes should use a sensible scale e.g. multiples of 1,2, 5 etc)

Identify how the following graphs could be improved

Graph 1



Graph 2



Forces and Motion

At GCSE you studied forces and motion and at A level you will explore this topic in more detail so it is essential you have a good understanding of the content covered at GCSE. You will be expected to describe, explain and carry calculations concerning the motion of objects. The websites below cover Newton's laws of motion and have links to these in action.

http://www.physicsclassroom.com/Physics-Tutorial/Newton-s-Laws

http://www.sciencechannel.com/games-and-interactives/newtons-laws-of-motion-interactive/

Sketch a velocity-time graph showing the journey of a skydiver after leaving the plane to reaching the ground.

Mark on terminal velocity.

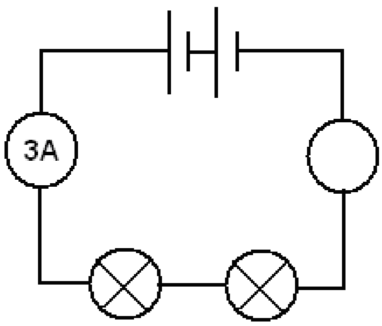
Electricity

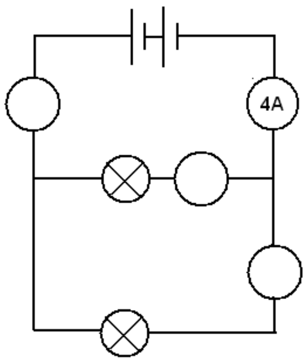
At A level you will learn more about how current and voltage behave in different circuits containing different components. You should be familiar with current and voltage rules in a series and parallel circuit as well as calculating the resistance of a device.

http://www.allaboutcircuits.com/textbook/direct-current/chpt-1/electric-circuits/

http://www.physicsclassroom.com/class/circuits

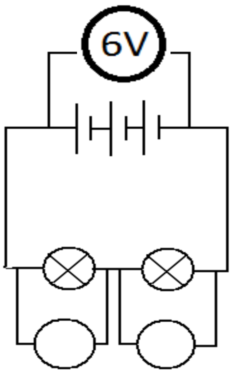
1a) Add the missing ammeter readings on the circuits below.

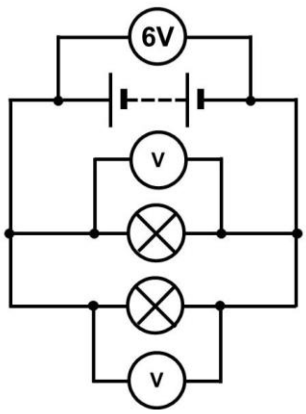




b) Explain why the second circuit has more current flowing than the first.

2) Add the missing potential differences to the following circuits





Waves

You have studied different types of waves and used the wave equation to calculate speed, frequency and wavelength. You will also have studied reflection and refraction.

Use the following links to review this topic.

http://www.bbc.co.uk/education/clips/zb7gkqt

https://www.khanacademy.org/science/physics/mechanical-waves-and-sound/mechanical-waves/v/introduction-to-waves

https://www.khanacademy.org/science/physics/mechanical-waves-and-sound/mechanical-waves/v/introduction-to-waves

1) Draw a diagram showing the refraction of a wave through a rectangular glass block. Explain why the ray of light takes this path.

2) Describe the difference between a longitudinal and transverse waves and give an example of each

3) Draw a wave and label the wavelength and amplitude

Pre-Knowledge Topics Answers:

Symbols and prefixes

1. 2400
2. 8 100 000
3. 326 000 000 000
4. 54.6
5. 240 000
6. 1.8×10^{-8}
7. 6.32×10^{-7}
8. 1.002
9. 5.11×10^{-5}
10. 1.1×10^{4}

Standard Form:

1. 2.53
2. 2.8
3. 7.7
4. 9.1
5. 1.872
6. 1.22
7. 2400
8. 35.05
9. 8 310 000
10. 600.2
11. 0.00015
12. 4300

Rearranging formulae

1. h= E/ (m x g)
2. I = Q/t
3. m = (2 x E)/v^2 or E/(0.5 x v^2)
4. v= √((2 x E)/m)
5. u = v – at
6. a = (v-u)/t
7. s = (v^2 – u^2) / 2a
8. u = √(v^2-2as)

Significant figures

1. 3.35
2. 40.7
3. 0.839
4. 1.02
5. 60.0
6. 0.809
7. 237
8. 3.4
9. 0.00330
10. 3343

Atomic Structure

contains protons, neutrons and electrons

Relative charge:

protons are positive (+1)

electrons are negative (-1)

neutrons are uncharged (0)

Relative mass:

proton 1

neutron 1

electron (about) 1/2000

protons and neutrons make up the nucleus

the nucleus is positively charged

electrons orbit the nucleus at a relatively large distance from the nucleus

most of the atom is empty space

nucleus occupies a very small fraction of the volume of the atom

most of the mass of the atom is contained in the nucleus

total number of protons in the nucleus equals the total number of electrons orbiting it in an atom

Recording data

Time should have a unit next to it

Length can be measured to the nearest mm so should be 10.0, 22.0 etc

Length 65 trial 2 is an anomaly and should have been excluded from the mean

All mean values should be to 2 decimal places

Mean of length 61 should be 6.99 (rounding error)

Graphs

Graph 1:

Axis need labels

Point should be x not dots

Line of best fit is needed

y axis is a difficult scale

x axis could have begun at zero so the y-intercept could be found

Graph 2:

y-axis needs a unit

curve of best fit needed not a straight line

Point should be x not dots

Forces and motion

Graph to show acceleration up to a constant speed (labelled terminal velocity). Rate of acceleration should be decreasing. Then a large decrease in velocity over a short period of time (parachute opens), then a decreasing rate of deceleration to a constant speed (labelled terminal velocity)

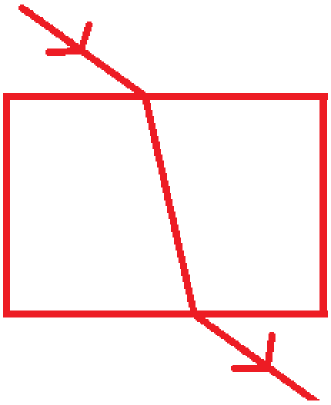
Electricity

1a) Series: 3A, Parallel top to bottom: 4A,2A,2A

b) Less resistance in the parallel circuit. Link to R=V/I. Less resistance means higher current.

2) Series: 3V, 3V, Parallel: 6V 6V

Waves



1) When light enters a more optically dense material it slows down and therefore bends towards the normal. The opposite happened when it leaves an optically dense material.

2) A longitudinal wave oscillates parallel to the direction of energy transfer (e.g. sound). A transverse waves oscillated perpendicular to the direction of energy transfer (e.g. light)

3)

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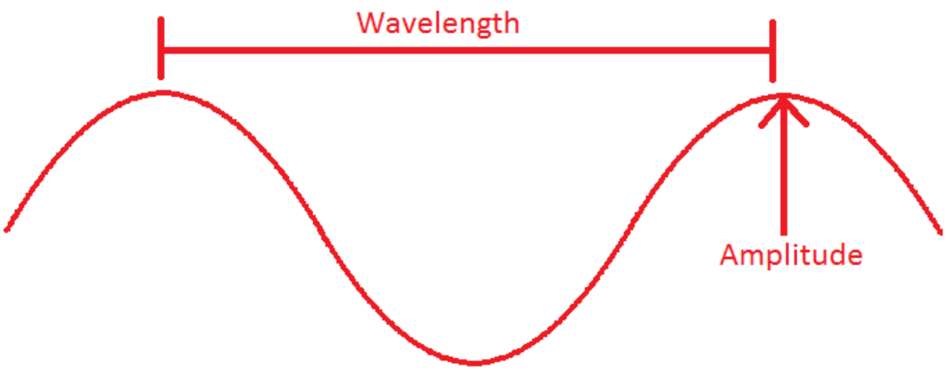


Using SI units

Specification references

- *3.1.1 Use of SI units and their prefixes*
- *M0.1 Recognise and make use of appropriate units in calculations*

Maths Skills for Physics references

- *1.1 Units and dimensions*

Learning objectives

After completing the worksheet you should be able to:

- show knowledge and understanding of base and derived SI units
- use equations to work out derived units
- use base units to check homogeneity of equations.

Introduction

Base quantities are measured in base units. These are units that are not based on other units. For example, mass is measured in kilograms and length is measured in metres. Other quantities have units which are derived from the base quantities. For example, the unit of density ($kg\,m^{-3}$) is derived from the kilogram and the metre.

The first example shows you how to use an equation to work out the unit of a derived quantity. The second example shows you how to check that an equation is homogeneous or, in other words, that its units are balanced.

Worked example

Question

What is the SI unit of speed?

Answer

Step 1

Identify the equation to use.

Speed is defined as: $\frac{\text{distance travelled}}{\text{time taken}}$

Step 2

Write the equation in terms of units.

The SI unit of speed is defined as: $\frac{\text{unit of distance travelled}}{\text{unit of time taken}}$

Step 3

Select the appropriate SI base units.

SI unit of distance = metre (m)

SI unit of time = second (s)

Step 4

Insert the SI base units into the equation.

SI unit of speed = $\frac{\text{metre (m)}}{\text{time taken (s)}}$ = metre per second = m s^{-1}

Question

1 Work out the missing units, unit symbols and names, equations, and quantities in this table. (*1 mark* for each correct answer)

Physical quantity	Equation used	Unit	Derived unit symbol and name
frequency	$\frac{1}{\text{time period}}$	**a**	Hz hertz
volume	length^3	**b**	–
acceleration	$\frac{\text{velocity}}{\text{time}}$	**c**	–
force	mass × acceleration	kg m s^{-2}	**d**
work and energy	force × distance	**e**	J joule
potential difference	$\frac{\text{energy}}{\text{electric charge}}$	J C^{-1}	**f**
electrical resistance	**g**	V A^{-1}	**h**
momentum	mass × velocity	**i**	–
impulse	force × time	**j**	–
k	$\frac{\text{force}}{\text{area}}$	**l**	Pa pascal
m	**n**	kg m^{-3}	–

Worked example

Question

Check that the equation: kinetic energy = $\frac{1}{2} m v^2$ is homogeneous.

Answer

Make sure you always state which side of the equation you are working on, left-hand side (LHS) or right-hand side (RHS).

Step 1

Start with the LHS. The unit of kinetic energy is the joule. Change this to base units.

LHS: $\text{J} = \text{N m} = \text{kg m s}^{-2} \times \text{m} = \text{kg m}^2 \text{s}^{-2}$

Step 2

Repeat Step 1 for the RHS.

RHS: units of $\frac{1}{2} m v^2$ are $\text{kg} \times (\text{m s}^{-1})^2 = \text{kg m}^2 \text{s}^{-2}$

(The constant, $\frac{1}{2}$, is a number with no units.)

Step 3

Don't forget to write your conclusion.

LHS = RHS so the equation is homogeneous.

We can't tell that there is a $\frac{1}{2}$ in the equation, so we cannot say that the equation is correct, only that it is homogeneous.

Questions

2 Use base units to show the equation $Q = I\,t$ for electric charge passing a point in time t, when the electric current is I, is homogeneous. (*1 mark*)

3 Use base units to show that the equation $P = I\,V$ is homogeneous, where I is electric current, V is voltage, and P is power measured in watts (W).
(Hint: $1\ \text{W} = 1\ \text{J s}^{-1}$) (*2 marks*)

4 The Earth's gravitational field strength, $g = 9.81\ \text{N kg}^{-1}$, is also sometimes given as the acceleration due to gravity, $g = 9.81\ \text{m s}^{-2}$. Show that these units are equivalent. (*1 mark*)

Maths skills links to other areas

You may also need to check equations are homogeneous wherever they are used in the specification – examples can be found in Chapter 7 *On the move*, and Topic 11.1 *Density*.

You can also use this method to help you decide whether you have remembered an equation correctly.

Answers

1 **a** s^{-1} (*1 mark*)

b m^3 (*1 mark*)

c $m\,s^{-2}$ (*1 mark*)

d N newton (*1 mark*)

e $kg\,m^2\,s^{-2}$ (allow N m, remind students that this is a derived unit) (*1 mark*)

f V volt (*1 mark*)

g $\frac{\text{voltage}}{\text{current}}$ (*1 mark*)

h Ω ohm (*1 mark*)

i $kg\,m\,s^{-1}$ (*1 mark*)

j N s (*1 mark*)

k pressure (*1 mark*)

l $N\,m^{-2}$ (*1 mark*)

m density (*1 mark*)

n $\frac{\text{mass}}{\text{volume}}$ (*1 mark*)

2 LHS: C = A s

RHS: A s

LHS = RHS (*1 mark*)

3 LHS: $W = J\,s^{-1} = kg\,m^2\,s^{-3}$ (*1 mark*)

RHS: $A \times V = A \times J\,C^{-1} = A \times kg\,m^2\,s^{-2} \times (A\,s)^{-1} = A \times kg\,m^2\,s^{-2} \times A^{-1} \times s^{-1} =$ $kg\,m^2\,s^{-3}$ (*1 mark*)

LHS = RHS

4 $N\,kg^{-1} = kg\,m\,s^{-2} \times kg^{-1} = m\,s^{-2}$ (*1 mark*)

Linear motion

Specification references

- *3.4.1.3*
- *M2.2 Change the subject of an equation, including non-linear equations*
- *M2.4 Solve algebraic equations, including quadratic equations*
- *M3.3 Understand that $y = mx + c$ represents a linear relationship*

Maths Skills for Physics references

- *3.3 Motion 3*

Learning outcomes

After completing the worksheet you should be able to:

- demonstrate an understanding of, and select and apply, the following equations:
  - $s = \frac{1}{2}(u + v)t$
  - $v = u + at$
  - $s = ut + \frac{1}{2}at^2$
  - $v^2 = u^2 + 2as$

  where s = displacement, u = initial velocity, v = final velocity, a = acceleration, and t = time
- understand that in free fall under gravity, objects fall with a constant acceleration, g, when air resistance is negligible.

Introduction

You do not have to memorise the equations of motion, sometimes referred to as the *suvat* equations. It is useful to know them, but always check the data sheet if you are not sure. The acceleration of free fall, g, will be provided on the data sheet so you should always use the value provided ($g = 9.81\ \text{m s}^{-2}$) unless the question tells you otherwise. You will lose marks for approximating it to $10\ \text{m s}^{-2}$.

When you are solving a problem:

- write down the values you know and the ones you want to calculate
- choose the equation and substitute all the values into it
- rearrange the equation and calculate the answer.

Worked example

Question

A student throws a ball vertically upwards at 5 m s^{-1}. When it comes down, the student catches it at the same point.

a Calculate the height the ball reaches.

b Calculate the length of time the ball is airborne.

Answer

a *Step 1*

Write down the values of *u*, *s*, *v*, *a*, and *t* that you know and those that you want to find.

You know $v = 0$ because as the ball rises it will slow down, until it comes to a stop and then it will start falling downwards. So when $v = 0$, the ball is at its maximum height.

Values are: $u = 5.0\ \text{m s}^{-1}$, $s = ?$, $v = 0$, $a = g = -9.81\ \text{m s}^{-2}$

Step 2

Use what you have written to choose the equation with just these variables.

In this case there is no *t*, so choose:

$$v^2 = u^2 + 2as$$

Step 3

Substitute the values, taking care to check the units (for example, in case distance is in km rather than m).

$0^2 = 5.0^2 + 2 \times -9.81 \times s$

$0 = 25 - 2 \times 9.81 \times s$

$0 = 25 - 19.62 \times s$

Step 4

Rearrange the equation so you can find *s*.

$19.62s = 25$

$s = \dfrac{25}{19.62} = 1.27\ \text{m} = 1.3\ \text{m}$ (2 significant figures)

b *Step 5*

At the starting point, $t = 0$ and $s = 0$. The ball travels upwards so that s increases to the maximum, then it starts falling and s decreases until $s = 0$ (because the ball has returned to its starting point). You want to find the time, t, at this point.

Write down the values of u, s, v, a, and t that you know and those that you want to find.

$u = 5.0\,\mathrm{m\,s^{-1}}$, $s = 0\,\mathrm{m}$, $a = g = -9.81\,\mathrm{m\,s^{-2}}$, $t = ?$

Step 6

Use what you have written to choose the equation with just these variables.

In this case there is no v, so choose:

$$s = ut + \frac{1}{2}at^2$$

Step 7

Substitute the values, taking care with the units.

$$0 = (5.0\,t) + \frac{1}{2}(-9.81)\,t^2$$

$$0 = 5.0\,t - \frac{1}{2}(9.81)\,t^2$$

$$0 = t\,(5.0 - 0.4905\,t)$$

(Notice that one solution is $t = 0$, because at the start when $t = 0$ the ball is at the starting point, $s = 0$. The other solution for t is at the end when the ball returns to the starting point, $s = 0$.)

The other solution, at the end, is given by:

$$5.0 = 0.4905\,t$$

Step 8

Calculate t.

$$t = \frac{5.0}{0.4905} = 1.02\,\mathrm{s} = 1.0\,\mathrm{s}\ \text{(2 significant figures)}$$

Questions

1 A racing car travelling at $13\,\mathrm{m\,s^{-1}}$ accelerates at $4.0\,\mathrm{m\,s^{-2}}$ for 9.0 s. What is its final speed? (*2 marks*)

2 A car travelling at $28\,\mathrm{m\,s^{-1}}$ slows down and stops in 75 m. Calculate the acceleration, assuming it is constant. (*2 marks*)

3 A stone is dropped down a dry well. It is heard to hit the bottom after 2.9 s. How deep is the well? (*2 marks*)

4 A rollercoaster accelerates from 0 to 27 m s^{-1} in 2.8 s. Calculate:

a the acceleration (*2 marks*)

b the distance travelled while accelerating. (*2 marks*)

5 A stone is dropped over the edge of a cliff and at the same time a small ball is fired vertically up in the air from the same height, with velocity 10 m s^{-1}, so that it falls and hits the beach next to the stone. The cliff is 100 m high. Calculate:

a the time for the ball to reach its maximum height (*2 marks*)

b the maximum height above the cliff reached by the ball (*2 marks*)

c the time for the ball to fall from this height to the beach (*2 marks*)

d the time for the stone to fall to the beach (*2 marks*)

e the time interval between the stone and the ball hitting the beach. (*2 marks*)

Maths skills links to other areas

You may also need to change the subject of an equation, solve algebraic equations, and understand that $y = mx + c$ represents a linear relationship, when plotting and interpreting suitable graphs from experimental results.

Answers

1 $v = u + at$

$v = (13\ \text{m s}^{-1}) + (4.0\ \text{m s}^{-2})(9.0\ \text{s})$ (*1 mark*)

$v = 49\ \text{m s}^{-1}$ (*1 mark*)

2 $v^2 = u^2 + 2as$

$0 = (28\ \text{m s}^{-1})^2 + 2a(75\ \text{m})$ (*1 mark*)

$a = -\frac{784}{150}\ \text{m s}^{-2} = -5.2\ \text{m s}^{-2}$ (*1 mark*)

3 $s = ut + \frac{1}{2}at^2$

Taking upwards as positive, $s = (0)(2.9\ \text{s}) + \frac{1}{2}(-9.81\ \text{m s}^{-2})(2.9\ \text{s})^2$ (*1 mark*)

$s = -41\ \text{m}$ (41 m downwards) (*1 mark*)

Allow: taking down as positive: $s = (0)(2.9\ \text{s}) + \frac{1}{2}(9.81\ \text{m s}^{-2})(2.9\ \text{s})^2$ and $s = 41\ \text{m}$

4 **a** $v = u + at$

$27\ \text{m s}^{-1} = 0 + a(2.8\ \text{s})$ (*1 mark*)

$a = \frac{27}{2.8}\ \text{m s}^{-2} = 9.6\ \text{m s}^{-2}$ (*1 mark*)

b $s = \frac{1}{2}(u + v)t$

$s = \frac{1}{2}(0 + 27) \times 2.8$ (*1 mark*)

$s = 37.8\ \text{m s}^{-1} = 38\ \text{m s}^{-1}$ (2 significant figures) (*1 mark*)

5 **a** Taking upwards as positive, ball thrown up:
$t = ?$, $u = 10.0\ \text{m s}^{-1}$, $v = 0$, $a = -g = -9.81\ \text{m s}^{-2}$

$v = u + at$

$0 = 10 - 9.81t$ (*1 mark*)

$t = \frac{10}{9.81} = 1.01\ \text{s} = 1.0\ \text{s}$ (2 significant figures) (*1 mark*)

b $s = ?$, $a = -g = -9.81\ \text{m s}^{-2}$, $u = 10\ \text{m s}^{-1}$, $v = 0$

$v^2 = u^2 + 2as$

$0 = (10)^2 - 2(9.81)s$ (*1 mark*)

$s = \frac{2 \times 105.1}{9.81} = 5.096\ \text{m} = 5.1\ \text{m}$ (2 significant figures) (*1 mark*)

c Total distance fallen = 100 m + 5.1 m so s = –105.1 m

$a = -g = -9.81\ \text{m s}^{-2}$, $u = 0$, t = ?

$$s = ut + \frac{1}{2}at^2$$

$$-105.1 = (0)t - \frac{1}{2}(9.81)t^2$$ (*1 mark*)

$$t^2 = \frac{2 \times 105.1}{9.81}$$

t = 4.63 s = 4.6 s (2 significant figures) (*1 mark*)

Allow: taking downwards as positive if it is clear.

d Stone dropped: u = 0, final s = –100 m, $a = -g = -9.81\ \text{m s}^{-2}$

$$s = ut + \frac{1}{2}at^2$$

$$-100 = (0)t + \frac{1}{2}(-9.81)t^2$$ (*1 mark*)

$$t^2 = \frac{200}{9.81}$$

t = 4.52 s = 4.5 s (2 significant figures) (*1 mark*)

Allow: taking downwards as positive if it is clear.

e Time for stone to reach beach from part **d** = 4.5 s

Time for ball to reach beach = time to reach maximum height + time to fall

= answer from part **a** + answer from part **c**

= 1.0 s + 4.6 s = 5.6 s (2 significant figures) (*1 mark*)

Time interval = 5.6 – 4.5 s = 1.1 s (*1 mark*)

The photon model

Specification references

- *3.2.1.3 Particles, antiparticles and photons*
- *3.5.1.2 Basics of electricity*
- *M0.2 Recognise and use expressions in decimal and standard form*
- *M2.4 Solve algebraic equations*

Maths Skills for Physics references

- *6.1 The photoelectric effect*

Learning objectives

After completing the worksheet you should be able to:

- calculate the energy of a photon in joules or in electronvolts given its frequency or wavelength
- calculate the frequency and wavelength of photons with energy given in joules or in electronvolts
- convert energies from joules to electronvolts, and vice versa.

Introduction

In some situations, electromagnetic radiation behaves as discrete packets of energy called photons. The energy of a photon, E, is directly proportional to its frequency.

$E = h f$

where f is the frequency of the electromagnetic radiation in Hz and $h = 6.63 \times 10^{-34}$ J s (the Planck constant).

You know that the wave equation is $c = f\lambda$.

Substituting $f = \frac{c}{\lambda}$ into the wave equation gives:

$$E = \frac{hc}{\lambda}$$

The energy of an individual photon or electron is very small, so the electronvolt (eV) is often used as a unit instead of the joule.

When an electron is accelerated though a potential difference (pd) of 1 V, it has an energy of 1 eV. Its energy in joules is calculated using the equation for the work done on the electron.

$W = Q V$, where Q is the charge on the electron e.

$\therefore W = e V$, where $e = 1.60 \times 10^{-19}$ C and $V = 1$ V

$W = (1.60 \times 10^{-19}\text{C}) \times (1\text{ V}) = 1.60 \times 10^{-19}$ J

So 1 electronvolt = 1.60×10^{-19} J, a very small amount of energy. (You can find this in your data sheet.)

If an electron is accelerated through 1000 V, its energy is 1000 eV = $1000 \times 1.6 \times 10^{-19}$ J = 1.6×10^{-16} J.

Worked example

Question

Electrons are accelerated in an X-ray tube by a pd of 25 kV.

a State the kinetic energy gained by the electrons in eV.

b Calculate the kinetic energy gained by the electrons in joules.

Answer

a *Step 1*

Use the definition of the eV (an electron accelerated through 1 V has energy 1 eV) to deduce the kinetic energy of an electron when accelerated through 25 kV.

V = 25 kV so E = 25 keV

b *Step 2*

Convert your answer to part **a** from eV to J. (Remember the joule is much bigger than the eV so your answer will be a much smaller number.)

$$25\text{ keV} = 25\text{ keV} \times (1.6 \times 10^{-19}\text{ J per eV})$$
$$= 25 \times 10^{3} \times 1.6 \times 10^{-19}\text{ J}$$
$$= 4.0 \times 10^{-15}\text{ J}$$

Questions

1 Electrons are accelerated in a discharge tube by a pd of 12 kV.

a State the kinetic energy gained by the electrons in eV. (*1 mark*)

b Calculate the kinetic energy gained by the electrons in joules. (*1 mark*)

2 The electrons hitting a screen have been accelerated through a vacuum tube and have each gained a kinetic energy of 6.4 keV. Calculate:

a the accelerating pd (*1 mark*)

b the kinetic energy gained by the electrons in joules. (*1 mark*)

3 A scientist requires a beam of electrons with energy 5.0×10^{-16} J. Calculate the accelerating pd required. (*1 mark*)

Worked example

Question

A visible light source has wavelength 488 nm.

a Calculate the frequency of the light.

b Calculate the energy of a photon in:

i joules

ii electronvolts.

Answer

a *Step 1*

Write down your known values, and substitute them into $c = f\lambda$.

$\lambda = 4.88 \times 10^{-7}$ m, $c = 3.0 \times 10^{8}$ m s^{-1}

$c = f\lambda$

3.0×10^{8} m s^{-1} $= f\,(4.88 \times 10^{-7}$ m)

Step 2

Rearrange the equation to calculate *f*.

$$f = \frac{3.0 \times 10^{8}\ \text{m s}^{-1}}{4.88 \times 10^{-7}\ \text{m}}$$

$$= 6.15 \times 10^{-14}\ \text{Hz}$$

b **i** *Step 3*

Substitute your value for *f* (from part **a**) into the equation $E = hf$ to calculate *E*.
$h = 6.63 \times 10^{-34}$ J s

$E = (6.63 \times 10^{-34}\ \text{J s}) \times (6.15 \times 10^{-14}\ \text{Hz})$

$= 4.08 \times 10^{-19}$ J

ii *Step 4*

Convert your answer from part **b i** to eV using 1 eV = 1.6×10^{-19} J, so

$1\ \text{J} = \frac{1}{1.6 \times 10^{-19}\ \text{J}}$ eV. (Remember your answer will be a bigger number than the number of joules because eVs are small.)

$$4.08 \times 10^{-19}\ \text{J} = \frac{4.08 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$= 2.55\ \text{eV}$$

Questions

4 Calculate the energy of a photon of red light with frequency 4.3×10^{14} Hz in:

a joules (*1 mark*)

b electronvolts. (*1 mark*)

5 Calculate the energy of a photon of violet light with wavelength 3.5×10^{-7} m in:

a joules (*1 mark*)

b electronvolts. (*1 mark*)

6 Calculate the energy of a photon of yellow light of wavelength 590 nm in:

a J (*1 mark*)

b eV (*1 mark*)

Worked example

Question

To ionise a hydrogen atom, a photon requires energy of 13.6 eV.

Calculate the wavelength of the electromagnetic radiation with this photon energy.

Answer

Step 1

Write down the equation and values you are using to calculate λ.

If you do not know which equation to use, start by writing out the variables you know, in this case *E*, *c*, and *h*, and then look to see which equation uses these variables.

$E = 13.6$ eV, $c = 3.0 \times 10^{8}$ m s^{-1}, $h = 6.63 \times 10^{-34}$ J s

$$E = \frac{hc}{\lambda}$$

Step 2

Change the energy in eV to an energy in J.

$E = 13.6 \text{ eV} = 13.6 \times 1.6 \times 10^{-19}$ J

$E = 2.18 \times 10^{-18}$ J

Step 3

Substitute the values into the equation.

$$2.18 \times 10^{-18}\text{ J} = \frac{(6.63 \times 10^{-34}\text{ J s})(3.0 \times 10^{8}\text{ m s}^{-1})}{\lambda}$$

Step 4

Rearrange the equation to make λ the subject.

$$\lambda = \frac{(6.63 \times 10^{-34}\,\text{J s})(3.0 \times 10^{8}\,\text{m s}^{-1})}{2.18 \times 10^{-18}\,\text{J}}$$

Step 5

Calculate λ.

$\lambda = 9.1 \times 10^{-8}$ m or 91 nm

Questions

7 A photon has energy 4.6×10^{-19} J. Calculate:

a its frequency (*1 mark*)

b its wavelength. (*1 mark*)

8 A photon has energy 10.21 eV. Calculate:

a its frequency (*2 marks*)

b its wavelength. (*1 mark*)

9 In a blue LED, a photon is emitted when an electron fills a positive hole (a gap left by a missing electron). The wavelength of the photon is 470 nm. Calculate the energy, in eV, transferred from the electron to the photon. (*2 marks*)

10 An electron is accelerated through a pd of 15 kV and strikes a metal target. If all the energy is transferred to a photon of electromagnetic radiation, deduce the wavelength of the photon emitted. (*2 marks*)

11 X-rays are required with a wavelength of 0.10 nm. Calculate the accelerating pd required for an X-ray tube to produce rays with a minimum wavelength of 0.10 nm. (*2 marks*)

Maths skills links to other areas

You will also need to calculate photon energy, and convert between joules and electronvolts in Topic 3.3 *Collisions of electrons with atoms*. You will need to be able to recognise and use expressions in decimal and standard form throughout the course.

Answers

1 **a** $V = 12\text{ kV}$ so $E = 12\text{ keV}$ (*1 mark*)

b $E = 12\text{ keV} \times (1.6 \times 10^{-19}\text{ J per eV})$

$E = 12 \times 10^{3} \times 1.6 \times 10^{-19}\text{ J}$

$= 1.9 \times 10^{-15}\text{ J}$ (*1 mark*)

2 **a** $E = 6.4\text{ keV}$ so $V = 6.4\text{ kV}$ (*1 mark*)

b $E = 6.4\text{ keV} \times (1.6 \times 10^{-19}\text{ J per eV})$

$E = 6.4 \times 10^{3} \times 1.6 \times 10^{-19}\text{ J} = 1.0 \times 10^{-15}\text{ J}$ (*1 mark*)

3 $E = 5.0 \times 10^{-16}\text{ J}$ so $E = \dfrac{5.0 \times 10^{-16}\text{ J}}{1.6 \times 10^{-19}\text{ J per eV}} = 3.1 \times 10^{3}\text{ eV}$

So $V = 3.1 \times 10^{3}\text{ V} = 3.1\text{ kV}$ (*1 mark*)

OR $V = \dfrac{E}{e}$

$= \dfrac{5.0 \times 10^{-16}\text{ J}}{1.6 \times 10^{-19}\text{ C}}$

$= 3.1 \times 10^{3}\text{ V} = 3.1\text{ kV}$ (*1 mark*)

4 **a** $E = (6.63 \times 10^{-34}\text{ J s}) \times (4.3 \times 10^{14}\text{ Hz}) = 2.9 \times 10^{-19}\text{ J}$ (*1 mark*)

b $E = \dfrac{2.9 \times 10^{-19}\text{ J}}{1.6 \times 10^{-19}\text{ J per eV}} = 1.8\text{ eV}$ (*1 mark*)

5 **a** $E = \dfrac{(6.63 \times 10^{-34}\text{ J s}) \times (3.00 \times 10^{8}\text{ m s}^{-1})}{3.5 \times 10^{-7}\text{ m}} = 5.7 \times 10^{-19}\text{ J}$ (*1 mark*)

b $E = \dfrac{5.7 \times 10^{-19}\text{ J}}{1.6 \times 10^{-19}\text{ J per eV}} = 3.6\text{ eV}$ (*1 mark*)

6 **a** $E = \dfrac{(6.63 \times 10^{-34}\text{ J s}) \times (3.00 \times 10^{8}\text{ m s}^{-1})}{590 \times 10^{-9}\text{ m}} = 3.4 \times 10^{-19}\text{ J}$ (*1 mark*)

b $E = \dfrac{3.4 \times 10^{-19}\text{ J}}{1.6 \times 10^{-19}\text{ J per eV}} = 2.1\text{ eV}$ (*1 mark*)

7 **a** $f = \dfrac{E}{h} = \dfrac{4.6 \times 10^{-19}\text{ J}}{6.63 \times 10^{-34}\text{ J s}} = 6.9 \times 10^{14}\text{ Hz}$ (*1 mark*)

b $\lambda = \dfrac{c}{f} = \dfrac{3.00 \times 10^{8}\text{ m s}^{-1}}{6.9 \times 10^{-14}\text{ Hz}} = 4.3 \times 10^{-7}\text{ m}$ (*1 mark*)

OR $\lambda = \dfrac{hc}{E} = \dfrac{(6.63 \times 10^{-34}\text{ J s})(3.00 \times 10^{8}\text{ m s}^{-1})}{4.6 \times 10^{-19}\text{ J}} = 4.3 \times 10^{-7}\text{ m}$ (*1 mark*)

8 **a** $10.21\text{ eV} = 10.21 \times 1.6 \times 10^{-19}\text{ J} = 1.63 \times 10^{-18}\text{ J}$ (*1 mark*)

$f = \dfrac{E}{h} = \dfrac{1.63 \times 10^{-18}\text{ J}}{6.63 \times 10^{-34}\text{ J s}} = 2.5 \times 10^{15}\text{ Hz}$ (*1 mark*)

b $\lambda = \dfrac{c}{f} = \dfrac{3.00 \times 10^{8}\,\text{m s}^{-1}}{2.5 \times 10^{15}\,\text{Hz}} = 1.2 \times 10^{-7}\,\text{m}$ (*1 mark*)

9 $E = \dfrac{hc}{\lambda} = \dfrac{(6.63 \times 10^{-34}\,\text{J s})(3.00 \times 10^{8}\,\text{m s}^{-1})}{470 \times 10^{-9}\,\text{m}} = 4.23 \times 10^{-19}\,\text{J}$ (*1 mark*)

$E = \dfrac{4.23 \times 10^{-19}\,\text{J}}{1.6 \times 10^{-19}\,\text{J per eV}} = 2.6\,\text{eV}$ (*1 mark*)

10 $V = 15\,\text{kV}$ so $E = 15 \times 10^{3} \times 1.6 \times 10^{-19}\,\text{J} = 2.4 \times 10^{-15}\,\text{J}$ (*1 mark*)

$\lambda = \dfrac{hc}{E} = \dfrac{(6.63 \times 10^{-34}\,\text{J s})(3.00 \times 10^{8}\,\text{m s}^{-1})}{2.4 \times 10^{-15}\,\text{J}} = 8.3 \times 10^{-11}\,\text{m}$ (*1 mark*)

11 $\lambda = 0.1\,\text{nm} = 0.1 \times 10^{-9}\,\text{m}$

$E = \dfrac{hc}{\lambda} = \dfrac{(6.63 \times 10^{-34}\,\text{J s})(3.00 \times 10^{8}\,\text{m s}^{-1})}{0.1 \times 10^{-9}\,\text{m}} = 1.99 \times 10^{-15}\,\text{J}$ (*1 mark*)

$E = \dfrac{1.99 \times 10^{-15}\,\text{J}}{1.6 \times 10^{-19}\,\text{J per eV}} = 1.2 \times 10^{4}\,\text{eV}$ so $V = 1.2 \times 10^{4}\,\text{V} = 12\,\text{kV}$ (*1 mark*)

OR use $W = QV$ so $E = eV$

$(1.99 \times 10^{-15}\,\text{J}) = (1.6 \times 10^{-19}\,\text{C})\,V$ (*1 mark*)

$V = \dfrac{1.99 \times 10^{-15}\,\text{J}}{1.6 \times 10^{-19}\,\text{C}} = 1.2 \times 10^{4}\,\text{V} = 12\,\text{kV}$ (*1 mark*)

Wave motion

Specification reference

- *3.3.1.1 Progressive waves*
- *3.3.1.2 Longitudinal and transverse waves*
- *M0.2 recognise and use expressions in decimal and standard form*
- *M1.1 use an appropriate number of significant figures*
- *M2.2 change the subject of an equation*

Maths Skills for Physics references

- *2.1 Graphs of waves*

Learning objectives

After completing the worksheet you should be able to:

- demonstrate and apply knowledge and understanding of the terms used to describe waves: displacement, amplitude, wavelength, period, frequency, and speed of a wave
- use the equation $f = \frac{1}{T}$
- use the equation $c = f\lambda$
- interpret an oscilloscope trace.

Introduction

A wave is produced when a vibrating source periodically disturbs nearby particles of a medium. This disturbance is passed from one particle to the next and this creates a wave pattern that travels through the medium.

Transverse waves have vibrations perpendicular (at 90°) to the direction of travel (the direction of propagation) of the wave.

Longitudinal waves have vibrations along the direction of (parallel to) the direction of propagation.

The **frequency**, *f,* at which each individual particle vibrates is equal to the frequency at which the source vibrates. Frequency is measured in hertz (Hz): 1 Hz = 1 complete oscillation per second.

The **period of vibration**, *T,* is measured in seconds and is the time taken for one complete oscillation, or the time for a wave to move one whole wavelength past a given point.

$$f = \frac{1}{T}$$

Amplitude, A, is the maximum displacement of a particle from its equilibrium position.

Wavelength, λ, is the shortest distance between two adjacent particles in the medium that have the same displacement and are moving in the same direction.

Wave speed, c, is the speed at which the wave travels through the medium.

The wave equation can be derived from the above definitions: $c = f\lambda$.

An oscilloscope can be used to determine the time period T of a wave. An oscilloscope shows a graph of potential difference against time. If each square on the time axis is 1 cm horizontally and the time base is set to 1 s cm^{-1} then each square represents a time interval of 1 s. The distance, in cm, of one complete wave on the time axis = time base setting × distance on time axis.

Worked example

Question

A transverse wave has amplitude 0.5 cm, a wavelength of 4.0 cm, and time period of 80.0 s. The displacement, s, is 0 cm at time $t = 0$ s, and distance travelled, $d = 0$ cm.

a Sketch a graph of:

- **i** displacement against distance travelled
- **ii** displacement against time.

b Calculate:

- **i** the frequency of the wave
- **ii** the wave speed of the wave.

Answer

a i *Step 1*

Draw a y-axis that extends from an amplitude of –0.5 cm to +0.5 cm, and an x-axis that allows for more than one wavelength – so it is longer than 4.0 cm.

Label your axes.

Step 2

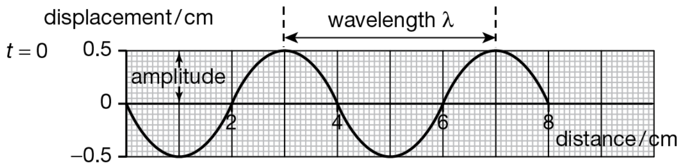
Plot points at the maximum (both positive and negative) and zero displacement for about two wavelengths.

Step 3

Draw a smooth curve through the points.

Step 4

Label the amplitude and the wavelength on the graph.



ii *Step 5*

Draw a *y*-axis that extends from an amplitude of –0.5 cm to +0.5 cm and an *x*-axis that allows for more than one time period – so it is longer than 80.0 s.

Label your axes.

Step 6

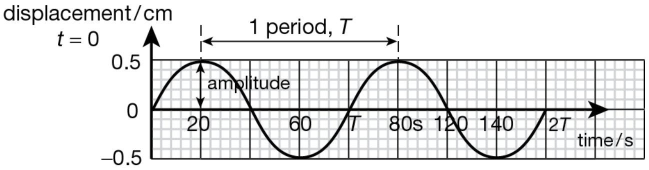
Plot points at the maximum (both positive and negative) and zero displacement for about two periods.

Step 7

Draw a smooth curve through the points.

Step 8

Label the amplitude and the period on the graph.



b i *Step 9*

Use the equation $f = \dfrac{1}{T}$ to calculate the frequency of the wave.

$$f = \frac{1}{T}$$

$$f = \frac{1}{80.0\,\text{s}}$$

$f = 0.0125\,\text{Hz}$

$= 0.013\,\text{Hz}$ (two significant figures)

ii *Step 10*

Substitute the wavelength and your value for f into the equation $v = f\lambda$.

$c = f\lambda$

$v = (0.0125\,\text{Hz})\,(4.0 \times 10^{-2}\,\text{m})$

Step 11

Calculate the wave speed, c.

$c = 5.0 \times 10^{-4}\,\text{m s}^{-1}$ **or** $5.0 \times 10^{-2}\,\text{cm s}^{-1}$

Questions

1 A wave has amplitude 8.0 mm, wavelength 1.5 m, and frequency 50 Hz.

a Sketch:

i a displacement–distance graph (*2 marks*)

ii a displacement–time graph. (*2 marks*)

b Calculate the wave speed. (*1 mark*)

2 A sound wave travels through a solid with a wavelength of 2.1 m and a frequency of 300 Hz. Calculate its speed. (*1 mark*)

3 A sound wave with frequency 250 Hz travels through a liquid with a wavelength of 5.7 m. Calculate its speed. (*1 mark*)

4 A musical note has frequency 512 Hz. Calculate its wavelength.
Speed of sound in air is 330 m s^{-1}. (*1 mark*)

5 A sound wave travels through a liquid with a wavelength of 1.5 m and a speed of 450 m s^{-1}.

Calculate:

a its frequency (*1 mark*)

b its period. (*1 mark*)

Worked example

Question

An electromagnetic wave travelling through the atmosphere has a wavelength of 1.5 km. Calculate its frequency.

Answer

Step 1

Remember that all electromagnetic waves travel with speed $c = 3.00 \times 10^{8}$ m s^{-1} in free space, and the difference in speed in the atmosphere is negligible.

Step 2

Substitute the values into the equation $c = f\lambda$.

$c = f\lambda$

$(3.0 \times 10^{8}\text{ m s}^{-1}) = f(1.5 \times 10^{3}\text{ m})$

Step 3

Rearrange the equation to make f the subject.

$$f = \frac{3.0 \times 10^{8}\text{ m s}^{-1}}{1.5 \times 10^{3}\text{ m}}$$

Step 4

Calculate f.

$f = 2.0 \times 10^{5}$ Hz

or $f = 200$ kHz

Questions

6 Calculate the frequency of light with wavelength 580 nm. (*1 mark*)

7 The hydrogen spectrum has a red line at 656 nm and the helium spectrum has a red line at 668 nm. Calculate the frequency of each line and hence find the difference in the frequencies. (*3 marks*)

8 In magnetic resonance imaging, the resonant or Larmor frequency of hydrogen nuclei in a magnetic field is about 64 MHz. What is the wavelength of the radio signal that gives this frequency? (*1 mark*)

Worked example

Question

Figure 1 shows a trace displayed on an oscilloscope. The time base is set to 2.0 ms cm^{-1}.

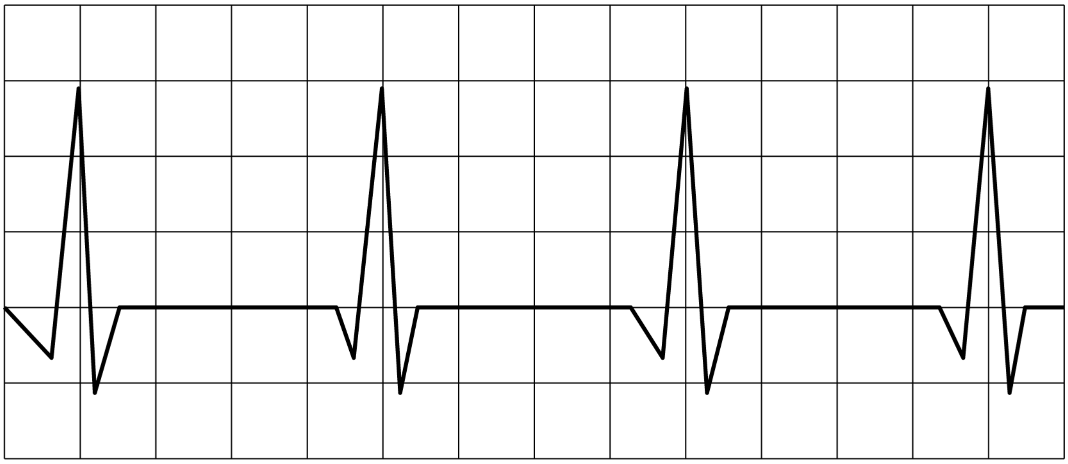


Figure 1

a Deduce the period of the wave.

b Calculate the frequency of the wave.

Answer

a *Step 1*

If the peaks line up with the grid lines, you can read the distance on the time axis easily. If not, use a ruler and measure the horizontal peak-to-peak distance on the paper to the nearest mm.

Horizontal peak-to-peak distance = 4.0 cm

Step 2

Set the ruler against the time axis with zero on the ruler lined up with a grid line on the time base, and read off the value corresponding to the length measured in Step 1.

4.0 cm is equivalent to 4.0 cm.

Step 3

Use the time base setting and the distance on the time axis to calculate the period.

$T = (2\ \text{ms cm}^{-1}) \times (4.0\ \text{cm})$

$= 8.0\ \text{ms}$

b *Step 4*

Calculate f from $f = \dfrac{1}{T}$.

$f = \dfrac{1}{T} = \dfrac{1}{8.0\ \text{ms}}$

$= 125\ \text{Hz}$

$= 130\ \text{Hz}$ (two significant figures)

Questions

9 Calculate the frequency of the wave in Figure 2 if the time base is set to:

a $5.0\ \text{ms cm}^{-1}$ (*2 marks*)

b $2.0\ \mu\text{s cm}^{-1}$ (*2 marks*)

c $100\ \text{ns cm}^{-1}$ (*2 marks*)

Each square represents 1 cm.

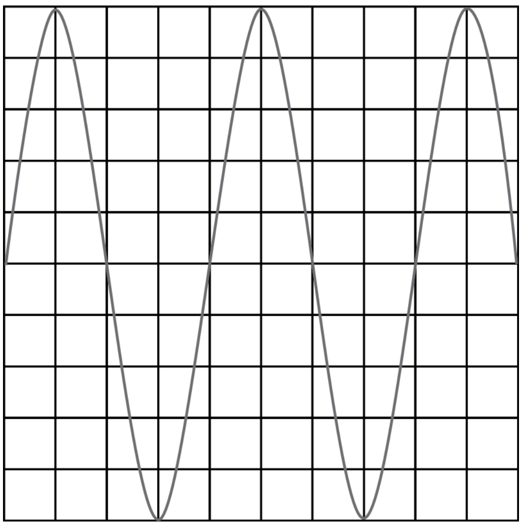
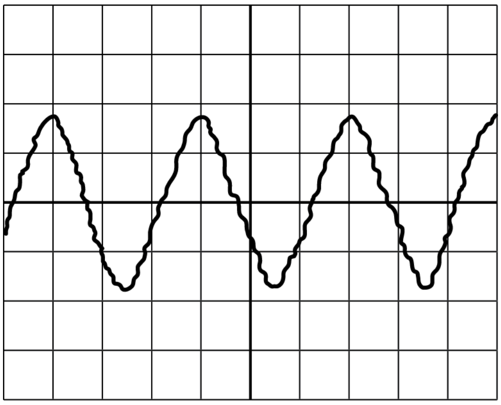


Figure 2

10 An electromagnetic wave is picked up by a detector, which produces an electrical signal. This signal is amplified and displayed on an oscilloscope screen. Each square represents 1 cm.



a The scale on the *y*-axis is $2.0\,\text{V cm}^{-1}$. Determine the amplitude of the electrical signal. (*1 mark*)

b The time base is set to $25\,\text{ns cm}^{-1}$. Determine the frequency of the signal and hence the wavelength of the electromagnetic wave. (*5 marks*)

c What type of electromagnetic wave is being detected? (*1 mark*)

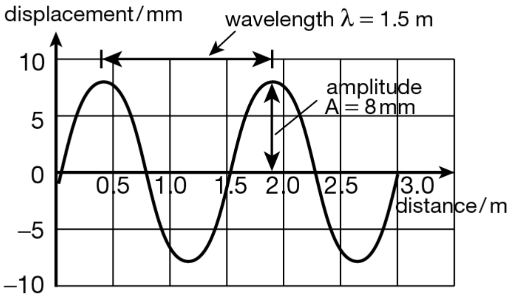
Maths skills links to other areas

It is important to always quote your final answers to an appropriate number of significant figures. During calculations you should always carry values to one more significant figure than your answer requires.

You will also need to recognise and use expressions in decimal and standard form throughout the course, including Topic 11.1 *Density*.

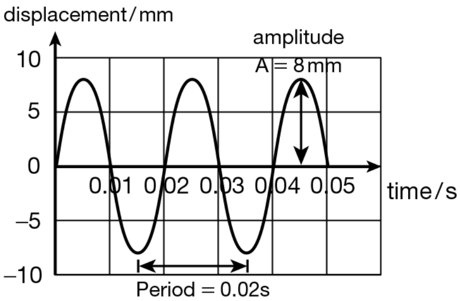
Answers

1 a i



(*1 mark correct amplitude, 1 mark correct wavelength*)

ii



1 mark for the correct amplitude and *1 mark* for the correct period

$(T = \frac{1}{f},\ T = \frac{1}{50\ \text{Hz}} = 0.02\ \text{s})$

b $c = f\lambda$

$c = (50\ \text{Hz}) \times (1.5\ \text{m})$

$= 75\ \text{m s}^{-1}$ (*1 mark*)

2 $c = f\lambda$

$c = (300\ \text{Hz}) \times (2.1\ \text{m})$

$= 630\ \text{m s}^{-1}$ (*1 mark*)

3 $c = f\lambda$

$c = (250\ \text{Hz}) \times (5.7\ \text{m})$

$= 1425\ \text{m s}^{-1}$

$= 1400\ \text{m s}^{-1}$ (two significant figures) (*1 mark*)

4 $\lambda = \frac{v}{f}$

$\lambda = \frac{330\ \text{m s}^{-1}}{512\ \text{Hz}}$

$= 0.64$ m **or** 64 cm (two significant figures) (*1 mark*)

5 a $f = \frac{c}{\lambda}$

$f = \frac{450\ \text{m s}^{-1}}{1.5\ \text{m}}$

$= 300\ \text{Hz}$ (*1 mark*)

b $T = \frac{1}{f}$

$T = \frac{1}{300\,\text{Hz}}$

$= 3.33 \times 10^{-3}\,\text{s}$

$= 3.3 \times 10^{-3}\,\text{s}$ or 3.3 ms (two significant figures) (*1 mark*)

6 $c = f\lambda$

$(3.00 \times 10^{8}\,\text{m s}^{-1}) = f\,(580 \times 10^{-9}\,\text{m})$

$f = \frac{3.00 \times 10^{8}\,\text{m s}^{-1}}{580 \times 10^{-9}\,\text{m}}$

$= 5.2 \times 10^{14}\,\text{Hz}$ (*1 mark*)

7 Hydrogen

$c = f\lambda$

$(3.00 \times 10^{8}\,\text{m s}^{-1}) = f\,(656 \times 10^{-9}\,\text{m})$

$f = \frac{3.00 \times 10^{8}\,\text{m s}^{-1}}{656 \times 10^{-9}\,\text{m}}$

$= 4.573 \times 10^{14}\,\text{Hz}$ (*1 mark*)

Helium

$c = f\lambda$

$(3.00 \times 10^{8}\,\text{m s}^{-1}) = f\,(668 \times 10^{-9}\,\text{m})$

$f = \frac{3.00 \times 10^{8}\,\text{m s}^{-1}}{668 \times 10^{-9}\,\text{m}}$

$= 4.491 \times 10^{14}\,\text{Hz}$ (*1 mark*)

Difference $= 4.573 \times 10^{14}\,\text{Hz} - 4.491 \times 10^{14}\,\text{Hz}$

$= 0.0820 \times 10^{14}\,\text{Hz}$

$= 8.20 \times 10^{12}\,\text{Hz}$ (three significant figures) (*1 mark*)

(Note that finding the difference in wavelength will not give the difference in frequency if substituted in the wave equation.)

8 $c = f\lambda$

$(3.00 \times 10^{8}\,\text{m s}^{-1}) = (64 \times 10^{6}\,\text{Hz})\,\lambda$

$\lambda = \frac{3.00 \times 10^{8}\,\text{m s}^{-1}}{64 \times 10^{6}\,\text{Hz}} = 4.7\,\text{m}$ (*1 mark*)

9 One wave is 4 cm on the screen.

a $T = 5.0\,\text{ms cm}^{-1} \times 4\,\text{cm}$

$= 20\,\text{ms}$ (*1 mark*)

$f = \frac{1}{20} \times 10^{-3}$

$= 50\,\text{Hz}$ (*1 mark*)

b $T = 2.0\ \mu s\ cm^{-1} \times 4\ cm$

$= 8\ \mu s$ (*1 mark*)

$f = \frac{1}{8} \times 10^{-6}$

$= 1.25 \times 10^{5}$ Hz

$= 130$ kHz (two significant figures) (*1 mark*)

c $T = 100\ ns\ cm^{-1} \times 4\ cm$

$= 400$ ns (*1 mark*)

$f = \frac{1}{400} \times 10^{-9}$

$= 2.5 \times 10^{6}$ Hz

$= 2.5$ MHz (*1 mark*)

10 a amplitude $= 1.8\ cm \times 2.0\ V\ cm^{-1}$

$= 3.6$ V (*1 mark*)

b $T = 25\ ns\ cm^{-1} \times 3.0\ cm$ (*1 mark*)

$= 7.5 \times 10^{-8}$ s (*1 mark*)

$f = \frac{1}{7.5 \times 10^{-8}\ s}$

$= 1.33 \times 10^{7}$ Hz

$= 1.3 \times 10^{7}$ Hz **or** 13 MHz (two significant figures) (*1 mark*)

$\lambda = \frac{c}{f}$

$\lambda = \frac{3 \times 10^{8}\ m\ s^{-1}}{1.33 \times 10^{7}\ Hz}$ (*1 mark*)

$= 22.6$ m

$= 23$ m (two significant figures) (*1 mark*)

c 23 m is in the radio frequency range – so the wave is a radio wave.
(Allow e.c.f. for wavelength.) (*1 mark*)

Determining uncertainty

Specification references

- *3.1.2 Limitation of physical measurements*
- *M0.3 Use ratios, fractions, and percentages*
- *M1.5 Identify uncertainties in measurements and use simple techniques to determine uncertainty when data are combined by addition, subtraction, multiplication, division, and raising to powers*

Maths Skills for Physics references

- *1.2 Uncertainties and significant figures*

Learning objectives

After completing the worksheet you should be able to:

- demonstrate knowledge and understanding of percentage errors and uncertainties
- evaluate absolute and percentage uncertainties
- determine uncertainty when data are combined by addition, subtraction, multiplication, division, and raising to powers.

Percentage uncertainties

Introduction

When something is measured there will always be a small difference between the measured value and the true value. There are several possible reasons for uncertainty in measurements, including the difficulty of taking the measurement, the precision of the measuring instrument (for example, due to the size of the scale divisions), and the natural variation of the quantity being measured. The word ‘uncertainty’ is generally used in preference to ‘error’, because ‘error’ implies something that is wrong – mistakes in making measurements should be avoided, and are not included in the uncertainty.

A measurement of 2.8 g is measured on a scale with divisions of 0.1 g. The true value could be anything from 2.85 g up to (but not including) 2.95 g, but the precision of the scale used does not allow us to measure the value to two decimal places. So we write the value as 2.8 (± 0.1) g. Here, 0.1 g is called the absolute uncertainty.

The percentage uncertainty in a measured value is calculated as shown below.

$$\text{percentage uncertainty} = \frac{\text{(absolute) uncertainty}}{\text{measured value}} \times 100\%$$

Worked example

Question

a The distance from **A** to **B** is carefully measured by stretching a 10-metre tape measure across the two points, taking a reading at both ends, and subtracting the larger value from the smaller value. The tape measure is marked in millimetre increments. The measured value is 7.500 m.

- **i** Deduce the absolute uncertainty in the measurement.
- **ii** Determine the percentage uncertainty in the measurement.

b The distance from **B** to **C** is measured as 6.5 m using a measuring wheel that gives measurements every 0.5 m. The wheel was reset to 0 and started rolling at point **B**.

- **i** Deduce the absolute uncertainty in the measurement.
- **ii** Determine the percentage uncertainty in the measurement.

c Calculate the absolute uncertainty in the total distance from **A** to **B** to **C**.

d Calculate the percentage uncertainty in the total distance from **A** to **B** to **C**.

Answer

a i *Step 1*

Consider the start point (**A**) and end point (**B**) of the measurement, and the scale division size. Due to the method used (a measurement was taken at both ends), there will be an uncertainty in the measurement both at the start point (**A**) and the end point (**B**).

The uncertainty in the measurement 1 mm at each end, giving a total uncertainty of 2 mm or 0.002 m.

Step 2

Write out the measurement with its absolute uncertainty. The uncertainty has the same unit as the measurement.

The distance **AB** is 7.500 ($\pm$ 0.002) m.

ii *Step 3*

Calculate the percentage uncertainty using the equation:

$$\text{percentage uncertainty} = \frac{\text{uncertainty}}{\text{measured value}} \times 100\%$$

$$\text{percentage uncertainty} = \frac{0.002}{7.500} \times 100\% = 0.027\% \text{ (to 2 significant figures)}$$

b i *Step 4*

This time there will be no uncertainty in the measurement at the start point (**B**), as the wheel was reset and no measurement was made at the start. The uncertainty and the end point (**C**) will be 0.5 m, as this is the precision of the measuring wheel.

Step 5

Write out the measurement with its absolute uncertainty.

The distance **BC** is 6.5 (± 0.5) m.

Step 6

$$\text{The percentage uncertainty} = \frac{0.5}{6.5} \times 100\%$$

$$= 7.7\% \text{ (to 2 significant figures)}$$

c *Step 7*

For the distance **ABC** the two measurements are added. The overall absolute uncertainty will be the sum of the individual absolute uncertainties.

uncertainty in **ABC** = uncertainty in **AB** + uncertainty in **BC**

= 0.002 m + 0.5 m

= 0.502 m (note, the 0.002 is fairly insignificant here)

d *Step 8*

To find the percentage uncertainty, first calculate the measured value of **ABC**.

ABC = 7.500 + 6.5 = 14 m

Step 9

Calculate the percentage uncertainty using the equation:

$$\text{percentage uncertainty} = \frac{\text{uncertainty}}{\text{calculated value}} \times 100\%$$

$$\text{percentage uncertainty} = \frac{0.502}{14} \times 100\%$$

$$= 3.6\% \text{ (to 2 significant figures)}$$

Questions

1 Write down these measurements with their absolute uncertainty.

- **a** 6.0 cm length measured with a ruler marked in mm (*1 mark*)
- **b** 0.642 mm diameter measured with a digital micrometer (*1 mark*)
- **c** 36.9 °C temperature measured with a thermometer which has a quoted accuracy of: '± 0.1 °C (34 to 42 °C), rest of range ± 0.2 °C'. (*1 mark*)

2 Calculate the percentage uncertainty in these measurements.

a 5.7 ± 0.1 cm (*1 mark*)
b 2.0 ± 0.1 A (*1 mark*)
c 450 ± 2 kg (*1 mark*)
d 10.60 ± 0.05 s (*1 mark*)
e 47.5 ± 0.5 mV (*1 mark*)
f 366 000 ± 1000 J (*1 mark*)

3 Calculate the absolute uncertainty in these measurements.

a 1200 W ± 10% (*1 mark*)
b 34.1 m ± 1% (*1 mark*)
c 330 000 Ω ± 0.5% (*1 mark*)
d 0.008 00 m ± 1% (*1 mark*)

4 Calculate the absolute and percentage uncertainty in the total mass of suitcases of masses x, y, and z.

$x = 23.3\ (\pm\ 0.1)$ kg, $y = 18\ (\pm\ 1)$ kg, $z = 14.7\ (\pm\ 0.5)$ kg (*2 marks*)

Combining uncertainties

Introduction

In a calculation, if several of the quantities have uncertainties then these will all contribute to the uncertainty in the answer. The following rules will help you calculate the uncertainty in your final answers.

- When quantities are added, the uncertainty is the sum of the *absolute* uncertainties.
- When quantities are subtracted, the uncertainty is also the sum of the *absolute* uncertainties.
- When quantities are multiplied, the *total percentage* uncertainty is the sum of the *percentage* uncertainties.
- When quantities are divided, the *total percentage* uncertainty is also the sum of the *percentage* uncertainties.
- When a quantity is raised to the power *n*, the *total percentage* uncertainty is *n* multiplied by the *percentage* uncertainty – for example, for a quantity x^2, total percentage uncertainty = 2 × percentage uncertainty in x.

Worked example

Question

A current of 2.8 (± 0.1) A passes through a kettle element. The mains power supply is 230 (± 12) V.

Calculate the power transferred, including its uncertainty.

Answer

Step 1

Calculate the power.

$P = IV$

$P = (2.8\text{ A}) \times (230\text{ V}) = 644\text{ W}$

Step 2

Calculate the percentage uncertainties.

The percentage uncertainty in current $= \dfrac{0.1}{2.8} \times 100\% = 3.57\%$

The percentage uncertainty in voltage $= \dfrac{12}{230} \times 100\% = 5.22\%$

The percentage uncertainty in power = 3.57% + 5.22% = 8.79% = 9% (to nearest %)

Step 3

Calculate the absolute uncertainty in the power.

The absolute uncertainty $= \dfrac{8.79}{100} \times 644\text{ W} = 57\text{ W}$

Step 4

State the answer with units.

Power = 644 (± 57) W

Questions

5 A piece of string 1.000 (± 0.002) m is cut from a ball of string of length 100.000 (± 0.002) m. Calculate the length of the remaining string and the uncertainty in this length. (*2 marks*)

6 A runner completes 100.0 (± 0.1) m in 18.6 (± 0.2) s. Calculate his average speed and the uncertainty in this value. (*2 marks*)

7 A car accelerates, with constant acceleration, from 24 (± 1) m s^{-1} to 31 (± 2) m s^{-1} in 9.5 (± 0.1) s. Calculate the acceleration. State your answer with its absolute uncertainty. (*3 marks*)

8 A cube has a mass of 7.870 (± 0.001) kg and sides of length 10.0 (± 0.1) cm. Give the value of the density of the cube. (*2 marks*)

9 In a Young's slits experiment, two slits that are very close together are illuminated, and on a distant screen an interference pattern of light and dark fringes is seen. The separation of the fringes can be used to calculate the wavelength of the light. In a demonstration of this experiment:

- the double slit separation, $s = 0.20$ (± 0.01) mm
- the distance from the slits to the screen, $D = 4.07$ (± 0.01) m
- the distance between two adjacent bright fringes $w = 12.0$ (± 0.05) mm.

The equation for calculating wavelength is $\lambda = \frac{ws}{D}$.

a Calculate:

i the wavelength, λ, of the light (*1 mark*)

ii the absolute uncertainty in the wavelength. (*2 marks*)

b The distance between 11 fringes (10 spaces) = 120.0 (± 0.05) mm. Using this value, calculate the new absolute uncertainty in the wavelength. (*2 marks*)

c Comment on whether the uncertainty in the wavelength could be significantly reduced by increasing the number of fringes measured to, for example, 20 or more. (*1 mark*)

Maths skills links to other areas

You may also need to calculate uncertainties when considering precision and accuracy of measurements and data, including margins of error, percentage errors, and uncertainties in apparatus.

Answers

1 **a** 6.0 (± 0.1) cm (*1 mark*)

b 0.642 (± 0.001) mm (*1 mark*)

c 36.9 (± 0.1) °C (*1 mark*)

2 **a** ± 1.8% (*1 mark*)

b ± 5% (*1 mark*)

c ± 0.44% (*1 mark*)

d ± 0.47% (*1 mark*)

e ± 1.1% (*1 mark*)

f ± 0.27% (*1 mark*)

3 **a** ± 120 W (*1 mark*)

b ± 0.3 m (*1 mark*)

c ± 2000 (or 1650) Ω (*1 mark*)

d ± 0.000 08 m (*1 mark*)

4 Absolute uncertainty is ± 1.6 kg (*1 mark*)

Percentage uncertainty is $\pm \frac{1.6}{56} \times 100\% = \pm 2.9\%$ (to 2 significant figures) (*1 mark*)

5 99.000 (*1 mark*) ± 0.004 m (*1 mark*)

6 Average speed = 5.376 m s^{-1} (*1 mark*)

Percentage uncertainty is 0.1% + 1.08% = 1.18%

Absolute uncertainty is ± 0.063 m s^{-1} (*1 mark*) (accept percentage or absolute uncertainty)

7 Change in speed = 7 (± 3) m s^{-1}

Acceleration = $\frac{7}{9.5}$ = 0.737 m s^{-2} (*1 mark*)

Percentage uncertainty = $\frac{3}{7} \times 100\% + \frac{0.1}{9.5} \times 100\%$

= 42.8% + 1.1%

= 43.9% (*1 mark*)

Absolute uncertainty = $\frac{43.9}{100} \times 7 = 3$

Acceleration with absolute uncertainty = 0.7 (± 0.3) m s^{-2} (*1 mark*)

8 Density = 7870 kg m^{-3} (1 mark)

Percentage uncertainty = $\frac{0.001}{7.870} \times 100\% + 3 \times \left(\frac{0.001}{0.1} \times 100\%\right)$

= 0.01% + 3%

= 3% to the nearest %

Density = 7900 (± 3% or ± 200) kg m^{-3} (*1 mark*) (accept percentage or absolute uncertainty)

9 a i $\lambda = \frac{ws}{D}$

$= \frac{(0.20 \times 10^{-3}\ \text{m}) \times (12.0 \times 10^{-3}\ \text{m})}{4.07\ \text{m}}$

$= 5.896 \times 10^{-7}$ m

$= 5.9 \times 10^{-7}$ m (2 significant figures) (*1 mark*)

ii % uncertainty in $\lambda = \frac{0.01 \times 100\%}{0.2} + \frac{0.05 \times 100\%}{12.0} + \frac{0.01 \times 100\%}{4.07}$

$= 5\% + 0.4\% + 0.2\%$

$= 5.6\%$

$= 6\%$ (to nearest %) (*1 mark*)

Absolute uncertainty $= 6\%$ of 5.9×10^{-7} m

$= 3.54 \times 10^{-8}$ m

$= 0.4 \times 10^{-7}$ m (1 significant figure) (*1 mark*)

b New % uncertainty $= 5\% + \frac{0.05 \times 100\%}{120.0} + 0.2\%$

$= 5\% + 0.04\% + 0.2\%$

$= 5\%$ to nearest % (*1 mark*)

Absolute uncertainty $= 5\%$ of 5.9×10^{-7} m

$= 2.95 \times 10^{-8}$ m

$= 0.3 \times 10^{-7}$ m (1 significant figure) (*1 mark*)

c The 5% uncertainty is due to the uncertainty in the slit separation *s*, so a further reduction in the uncertainty in *x* would not reduce the total uncertainty. (*1 mark*)

Allow errors carried forward from results in parts **a** and **b**.

Using scalars and vectors

Specification references

- *3.4.1.1*
- *M0.6 Use calculators to handle sin x, cos x, and tan x when x is expressed in degrees or radians*
- *M4.2 Visualise and represent 2D and 3D forms*
- *M4.4 Use Pythagoras' theorem and the angle sum of a triangle*
- *M4.5 Use sin, cos, and tan in physical problems*

Maths Skills for Physics references

- *3.1 Motion 1*
- *3.4 Forces*
- *3.5 Resolving forces*

Learning outcomes

After completing the worksheet you should be able to:

- show and apply knowledge and understanding of scalar and vector quantities
- solve problems involving vector addition and subtraction
- use a vector triangle to determine the resultant of any two coplanar vectors.

Introduction

Scalar quantities have magnitude but no direction. For example, speed, distance, and time are all scalar quantities.

Vector quantities have magnitude and direction. Velocity is a vector quantity: it is speed in a certain direction. When we calculate velocity, *v*, we need to know the displacement, *s*. Displacement is also a vector: it is the distance travelled in a certain direction. The direction of a vector is sometimes indicated by giving an angle to a reference direction, for example, north. Sometimes a vector has a positive direction and a negative direction, in this case, the negative direction is opposite to the positive direction.

Acceleration is the rate of change of velocity, not of speed. This means that it is a vector. Consider an object moving in a circle at constant speed. Its direction is constantly changing, which means its velocity is changing. Therefore, it is accelerating.

When you add or subtract vectors, you must take the direction into account. Figure 1 shows that walking 3 m from **A** to **B**, and then turning through 30° and walking 2 m to **C**, has the same effect as walking directly from **A** to **C**. $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are vectors and are shown by a single arrowhead on a vector diagram. $\overrightarrow{AC}$ is the resultant vector, shown by the double arrowhead.

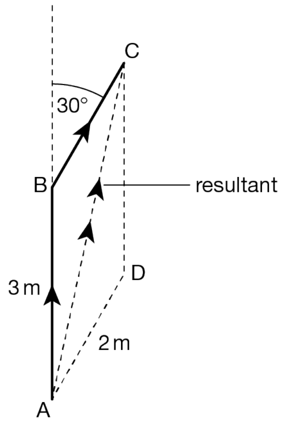
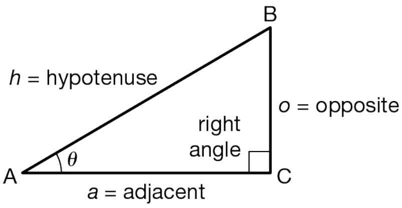


Figure 1

To combine any two vectors, we can draw a triangle like **ABC** in Figure 1, where the lengths of the sides represent the magnitude of the vector (for example, forces of 30 N and 20 N). The third side of the triangle shows us the magnitude and direction of the resultant force. Careful drawing of a scale diagram allows us to measure these.

Notice that if the vectors are combined by drawing them in the opposite order (**AD** and **DC** in Figure 1) these are the other two sides of a parallelogram and give the same resultant. If you draw both vectors so they start from point **A**, their resultant will be the diagonal of the parallelogram.

In solving problems with triangles, remember the angles in a triangle add up to 180°. In a right-angled triangle this means the other two angles add up to 90°.



For a right-angled triangle as shown, Pythagoras' theorem says:

$h^2 = o^2 + a^2$

Also:

$\sin\theta = \frac{o}{h}$, $\cos\theta = \frac{a}{h}$, $\tan\theta = \frac{o}{a}$ (some people remember: soh cah toa)

Worked example

Question

A sub-atomic particle experiences two forces at right angles, one of 2.0×10^{-15} N the other 3.0×10^{-15} N. Calculate the resultant force on the particle.

Answer

Step 1

Draw a diagram showing the two forces on the particle.

You can either draw the two forces acting on the particle at the same point, with the resultant as the diagonal of the rectangle formed, or consider the forces acting one after the other with the resultant as the third side of the triangle. Figure 2 shows both of these diagrams. In each the resultant is represented by *F*.

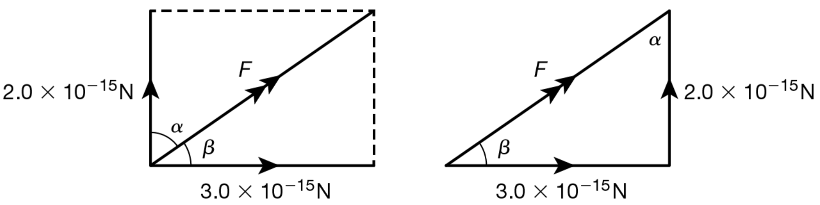


Figure 2

Step 2

Use Pythagoras' theorem to calculate the magnitude of the resultant, *F*.

$$F^2 = (2.0 \times 10^{-15}\text{ N})^2 + (3.0 \times 10^{-15}\text{ N})^2$$

$$= (4.0 \times 10^{-30} + 9.0 \times 10^{-30})\ \text{N}^2$$

Step 3

Don't forget to take the square root of your answer.

$$F = \sqrt{13 \times 10^{-30}}\ \text{N}$$

Step 4

Write your answer to the same number of significant figures as the question and with the correct units.

$$F = 3.6 \times 10^{-15}\text{ N}$$

Step 5

Either calculate the angle to the vertical using tan α, or calculate the angle to the horizontal using tan β.

$$\tan \alpha = \frac{3.0 \times 10^{-15}\text{ N}}{2.0 \times 10^{-15}\text{ N}} = 1.5$$

Use your calculator to find α.

$$\alpha = \tan^{-1} 1.5 = 56°$$

Or:

$$\tan \beta = \frac{2.0 \times 10^{-15}\text{ N}}{3.0 \times 10^{-15}\text{ N}} = 0.67$$

Use your calculator to find β.

$\beta = \tan^{-1} 0.67 = 34°$

Step 6

Don't forget to write out your final answer.

If you used angle α:

Resultant force = 3.6×10^{-15} N at 56° to the 2.0×10^{-15} N force

Or, if you used angle β:

Resultant force = 3.6×10^{-15} N at 34° to the 3.0×10^{-15} N force

Questions

1 Divide these quantities into vectors and scalars. (*3 marks*)

density
electric charge
electrical resistance
energy
field strength
force
friction
frequency
mass
momentum
power
voltage
volume
weight
work done

2 Divide these data into vectors and scalars. (*2 marks*)

3 m s^{-1}

$+20 \text{ m s}^{-1}$

100 m NE

50 km

–5 cm

10 km S30°W

3 **a** Sketch a vector triangle showing a force of 3.0 N and a force of 4.0 N acting at right angles, and the resultant of the two vectors. (*2 marks*)

b Determine the magnitude and direction of the resultant force. (*2 marks*)

4 Find the resultant force of a 5.0 N and 12.0 N force acting at right angles. (*2 marks*)

5 A ship is cruising at 9.4 m s^{-1} and a boy runs across the deck, at right angles to the direction of the ship, at 3.0 m s^{-1}. Calculate his resultant velocity. (*2 marks*)

6 An aircraft flies east at a speed of 53 m s^{-1}. The wind is blowing from the north at a constant 16 m s^{-1}. What is the resultant velocity of the aircraft? (*2 marks*)

7 Two tugboats are towing a ship in a straight line. Tug A is pulling with a force of 50 kN at 60° to the direction in which the ship is moving. Tug B is pulling at 30° to the direction in which the ship is moving. Draw a sketch and then calculate the magnitude of:

a the resultant force on the ship (*1 mark*)

b the force from tug B. (*1 mark*)

Maths skills links to other areas

You may need to do similar calculations and draw vector diagrams in Topic 7.7 *Projectile motion 1*, and Topic 7.8 *Projectile motion 2*.

Answers

1 Scalars: density, electric charge, electrical resistance, energy, frequency, mass, power, voltage, volume, work done

Vectors: field strength, force, friction, momentum, weight

Award *3 marks* for all correct, *2 marks* for one wrong, *1 mark* for two wrong, and *0 marks*
for three or more wrong.

2 Scalars: $3\ \text{m s}^{-1}$, 50 km

Vectors: $+20\ \text{m s}^{-1}$, 100 m NE, –5 cm, 10 km S30°W

Award *2 marks* for all correct, *1 mark* for one wrong, and *0 marks* for two or more wrong.

3 **a** The two sides representing 3.0 N and 4.0 N meet at right angles. One has an arrowhead towards the right angle and the other has an arrowhead away from the right angle. (*1 mark*)

The resultant is the hypotenuse (opposite side to the right angle) and has a double arrow. The direction is such that it is an equivalent alternative route – see Figure 1
on the student sheet. (*1 mark*)

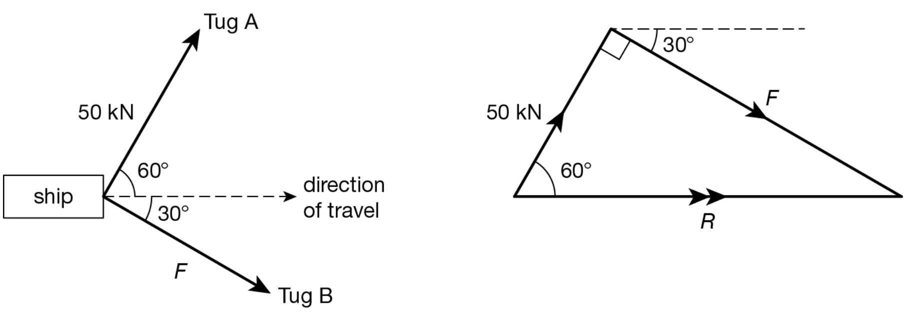
b $R = \sqrt{[(5.0\ \text{N})^2 + (12.0\ \text{N})^2]}$ = 5.0 N (*1 mark*) at an angle $\tan^{-1}(3.0 / 4.0) = 37°$ to the 4.0 N force (*1 mark*)

4 $R = \sqrt{[(5.0\ \text{N})^2 + (12.0\ \text{N})^2]}$ = 13 N (*1 mark*) at an angle $\tan^{-1}(5.0 / 12.0) = 23°$ to the 12.0 N force (*1 mark*)

5 $v_R = \sqrt{[(9.4\ \text{m s}^{-1})^2 + (3.0\ \text{m s}^{-1})^2]} = 9.9\ \text{m s}^{-1}$ (*1 mark*) at an angle $\tan^{-1}(3.0 / 9.4) =$ 18° to
the direction of the ship (*1 mark*)

6 $v_R = \sqrt{[(53\ \text{m s}^{-1})^2 + (16\ \text{m s}^{-1})^2]} = 55\ \text{m s}^{-1}$ (*1 mark*) at an angle $\tan^{-1}(16 / 53) =$ 17° south
of east (*1 mark*)

7



a $R = \dfrac{50\ \text{kN}}{\cos 60°} = 100\ \text{kN}$ (*1 mark*)

b $F = (100\ \text{kN}) \cos 30° = 87\ \text{kN}$ (*1 mark*)

Triangles of forces

Specification references

- *3.4.1.1*
- *3.4.1.2*
- *M4.1 Use angles in regular 2D and 3D structures*
- *M4.2 Visualise and represent 2D and 3D forms including 2D representations of 3D objects*
- *M4.4 Use Pythagoras' theorem and the angle sum of a triangle*

Maths Skills for Physics references

- *3.4 Forces*
- *3.5 Resolving forces*

Learning outcomes

After completing the worksheet you should be able to:

- demonstrate knowledge and understanding of the conditions for equilibrium under the action of forces
- sketch a triangle of forces for an object in equilibrium
- make a scale drawing of a triangle of forces and apply this to deduce the magnitude and direction of a force
- use a triangle of forces to calculate the magnitude and direction of a force.

Conditions for equilibrium

Introduction

An object is in equilibrium when the resultant force on it is zero.

The pendulum bob in Figure 1 will not stay in its current position if you remove your finger, because the tension, **T**, and weight, **W**, have a resultant force, **R**, that will return the string to a vertical position. (You can find more about resultant forces in '6.1 Calculation sheet: Using scalars and vectors'.) When the force from the finger, **F**, is equal and opposite to the resultant, **R**, then the bob is in equilibrium and the three forces, **T**, **F**, and **W**, form a triangle of forces.

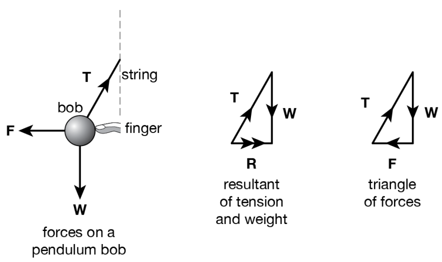
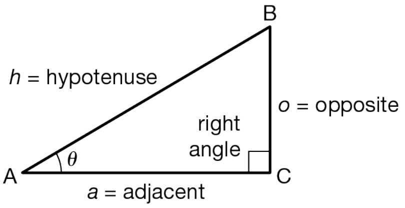


Figure 1

If three forces are acting on an object and it is in equilibrium, then the forces will always form a closed triangle of forces. By drawing the triangle of forces, you can find the magnitude and direction of one of the forces if you know the other two. In solving problems with triangles, remember the angles in a triangle add up to 180°. In a right-angled triangle this means the other two angles add up to 90°.



For a right-angled triangle as shown, Pythagoras' theorem says:

$h^2 = o^2 + a^2$

Also:

$\sin\theta = \frac{o}{h}$, $\cos\theta = \frac{a}{h}$, $\tan\theta = \frac{o}{a}$ (some people remember: soh cah toa)

Worked example

Question

A box weighing 100 N is on a slope with angle 30° to the horizontal. Friction prevents the box sliding down the slope.

a Sketch a triangle of forces for the forces acting on the box.

b Draw a scale diagram and deduce the magnitude of the friction force.

Answer

a *Step 1*

First sketch the situation: the box on the slope of 30° to the horizontal, the weight of the box, the normal contact force perpendicular to the slope, and the friction parallel to and up the slope.

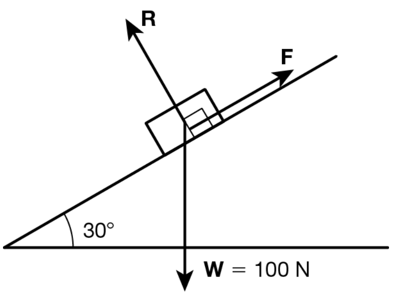


Figure 2 *Sketch of the situation*

Step 2

Decide how to draw the triangle of forces. The three forces must be in the directions shown in Figure 2, and you must put them together in an order so that they all follow round, in a clockwise, or an anticlockwise, direction. You cannot have one going in the opposite direction.

Sketch the triangle, and mark the angle, or angles, you know – remember that angles in a triangle add to 180°.

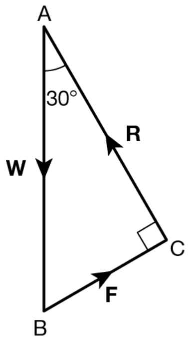


Figure 3 *Sketch of the triangle of forces*

b *Step 3*

Decide on a scale – make your diagram as big as you can, because it will be more accurate. Make sure you have a sharp pencil, a straight ruler – transparent ones are best as you can see what you are covering – and a protractor.

For example: Scale: 1 cm = 10 N

Step 4

Draw a vertical line and mark **AB**, which is 10 cm, representing the weight of 100 N.

Step 5

Use the protractor to draw a line that starts at **A** and is at 30° to **AB**. Make sure it is long enough (use your sketch, Figure 3, to guide you).

Step 6

Use the protractor to draw a line that starts at **B** and is at 60° to **AB**. Make sure it crosses your line from step 5. The point at which they cross is point **C**. You can check you have drawn it accurately because the angle at **C** should be 90°.

Step 7

Label the lines and add arrowheads to show the directions of the forces.

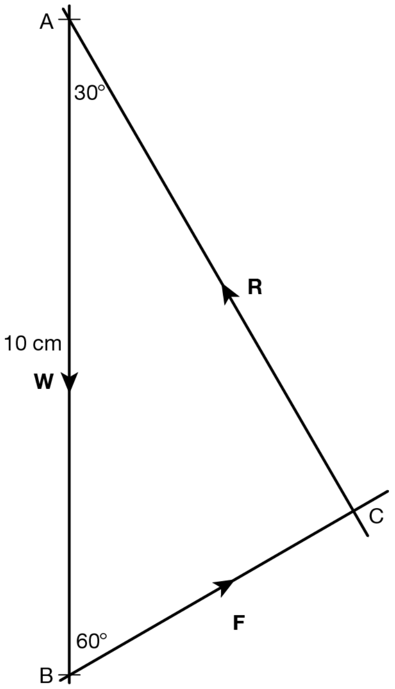


Figure 4 *Scale diagram of the triangle of forces*

Step 8

Measure the length of **BC**.

BC = 5.0 cm

Step 9

Use your scale to work out the magnitude of the friction.

Friction = 5.0 cm × 10 N per cm = 50 N

Questions

1 There are three forces on the jib of a tower crane. These are the tension in the cable **T**, the weight **W**, and a third force **F** which acts at the point **X**. See Figure 5.

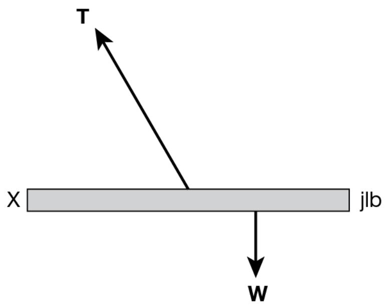


Figure 5

The crane is in equilibrium. Sketch the triangle of forces. (*1 mark*)

2 Two forces of 5 kN are towing a boat, as shown in Figure 6.

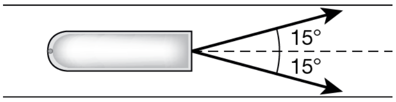


Figure 6

The boat is travelling at constant speed. Sketch a triangle of forces showing the towing forces and the drag force acting on the boat. (*1 mark*)

3 The three forces in Figure 7 are in equilibrium.

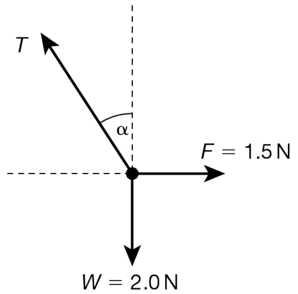


Figure 7

a Sketch a triangle of forces. (*1 mark*)

b Draw a scale diagram and deduce the magnitude of **T** and the angle α. (*3 marks*)

4 A climber of weight 600 N is walking down a vertical cliff face using a rope which makes an angle of 20° to the cliff.

a Sketch the free-body diagram when one leg is in contact with the cliff and is horizontal. (*1 mark*)

b Draw the triangle of forces. (*1 mark*)

c Find the magnitude of the tension in the rope by scale drawing. (*1 mark*)

Strings and cables

Introduction

Strings or cables are often used to exert a force to keep an object in position.

Remember that a string or cable can be in tension, but not in compression. If it is in tension, the force pulls on the objects it is attached to.

You should know that the tension is the same at every point in the cable, because otherwise the cable would break, so it is always the same at both ends.

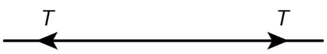


Figure 8 *A cable in tension*

Worked example

Question

1 A lamp hangs from three cables that are tied as shown in Figure 9.

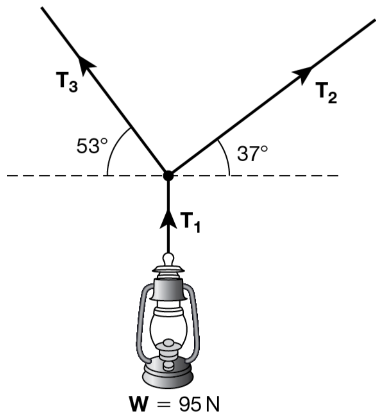


Figure 9

The lamp and the knot in the cable are both in equilibrium.

a Draw free-body diagrams for the lamp and the knot in the cable.

b Deduce the magnitude of the tension $\mathbf{T_1}$.

c Sketch a triangle of forces for the knot in the cable.

d Calculate the magnitudes of the tensions $\mathbf{T_2}$ and $\mathbf{T_3}$.

Answer

a *Step 1*

Sketch the free-body diagram for the lamp, showing all the forces on the lamp – the weight and the tension in the cable $\mathbf{T_1}$.

Step 2

Sketch the free-body diagram for the knot in the cable, showing the tensions in the three cables.

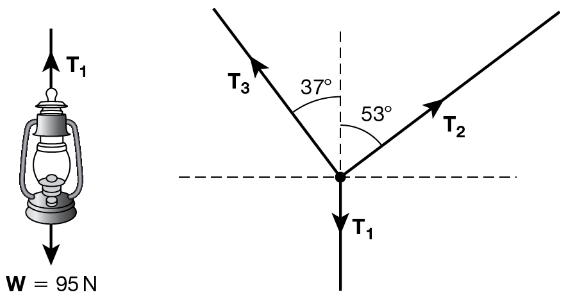


Figure 10 *The free-body force diagrams*

b *Step 3*

If the lamp is in equilibrium, the two forces on it must be equal and opposite.

For the lamp: magnitude of $\mathbf{T_1}$ = magnitude of $\mathbf{W}$ = 95 N

c *Step 4*

Sketch the triangle of forces for the three tension forces. Remember to keep their directions as they are in the free-body diagram (Figure 10), and draw them in order so that they all follow round in an anti-clockwise direction.

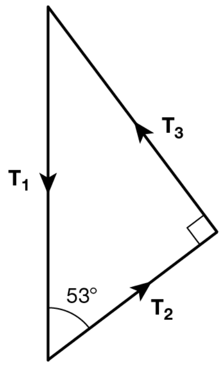


Figure 11 *The triangle of forces for the knot*

Step 5

Calculate the third angle and mark it on the diagram.

Third angle = 180° – (90° + 52°) = 37°

$\mathbf{T_1}$ is downwards, $\mathbf{T_2}$ is at 53° to $\mathbf{T_1}$, and $\mathbf{T_3}$ is at (37° + 53°) to $\mathbf{T_2}$.
Notice (37° + 53°) = 90°.

d *Step 6*

The triangle of forces is a right-angled triangle. Use cos 53° or sin 37° to calculate the magnitude of $\mathbf{T_2}$.

Magnitude of $\mathbf{T_2}$ = (95 N) cos 53° = 57 N (2 significant figures)

Or:

Magnitude of $\mathbf{T_2}$ = (95 N) sin 37° = 57 N (2 significant figures)

Step 7

Use cos 37° or sin 53° to calculate the magnitude of $\mathbf{T_3}$.

Magnitude of $\mathbf{T_3}$ = (95 N) cos 37° = 76 N (2 significant figures)

Or:

Magnitude of $\mathbf{T_3}$ = (95 N) sin 53° = 76 N (2 significant figures)

Questions

5 The three strings in Figure 12 are in tension and in equilibrium.

a Sketch a triangle of forces. (*1 mark*)

b Calculate the tension in each string. (*2 marks*)

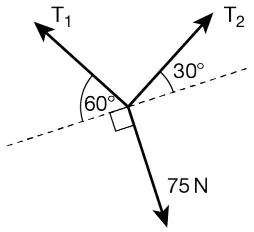


Figure 12

6 A sign of mass 50 kg is supported by a wire and a rod as shown in Figure 13.

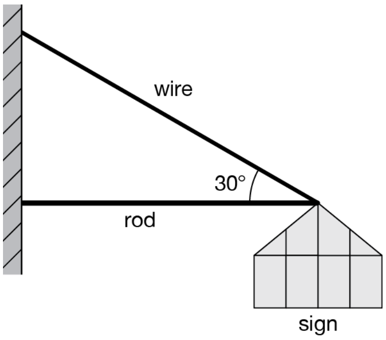


Figure 13

a Sketch the free-body diagram for the sign. (*1 mark*)

b Sketch the triangle of forces for the sign. (*1 mark*)

c Calculate the tension in the wire. (*3 marks*)

7 Figure 14 shows a ball of weight 200 N which is held in place by two cables **AB** and **BC**.

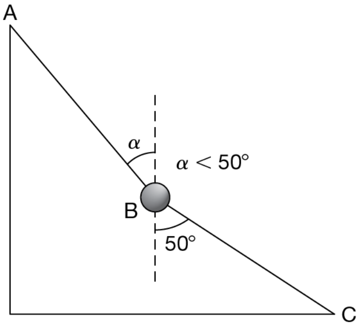


Figure 14

a Draw a free-body diagram. Label the tension in cable **AB** as $\mathbf{T_1}$ and the tension in cable **BC** as $\mathbf{T_2}$. (*1 mark*)

b Determine the magnitudes of $\mathbf{T_2}$ and $\mathbf{T_1}$ when $\alpha = 25°$, by drawing a scale diagram of the triangle of forces, or by calculation. (*3 marks*)

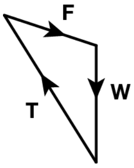
c Explain why the ball cannot be in equilibrium if $\alpha = 50°$ and **BC** stays at 50° to the vertical. (*1 mark*)

Maths skills links to other areas

For more information on triangles of forces, see Chapter 6 *Forces in equilibrium*.

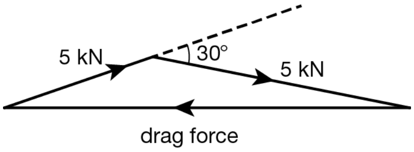
Answers

1



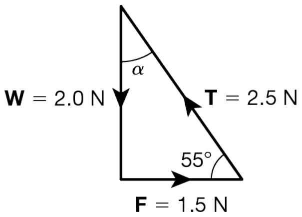
(*1 mark*)

2



(*1 mark*)

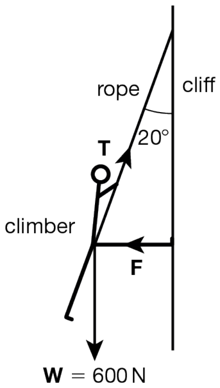
3 **a**



(*1 mark*)

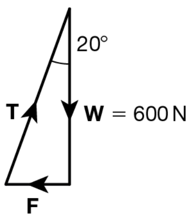
b Magnitude of **T** = 2.5 N, $\alpha = 35°$. (*1 mark* for a scale drawing, e.c.f. wrong triangle, *1 mark* for correct **T**, and *1 mark* for correct α. No marks for **T** and α if found by calculation.)

4 **a** Students should draw the three forces at the midpoint of the person, as shown.



(*1 mark*)

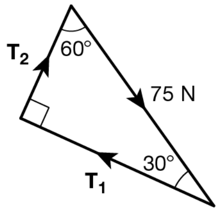
b



(*1 mark*)

c Magnitude of **T** = 640 N (*1 mark*)

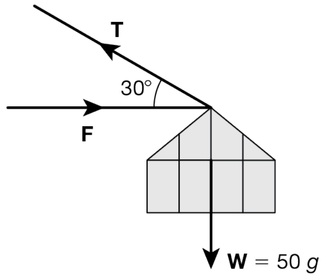
5 a



(*1 mark*)

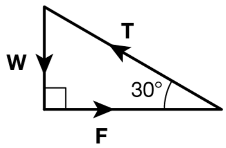
b Tensions are 75 N (given), $\mathbf{T_1}$ = 75 cos 30° = 65 N (*1 mark*), $\mathbf{T_2}$ = 75 sin 30° = 38 N (*1 mark*).

6 a



(*1 mark*)

b

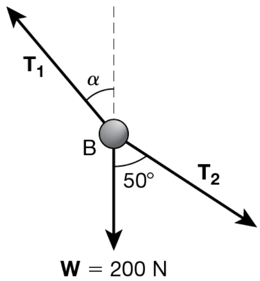


(*1 mark*)

c Magnitude of **W**: $W = 50g = 50\text{ kg} \times 9.81\text{ m s}^{-2} = 490.5\text{ N}$ (*1 mark*)

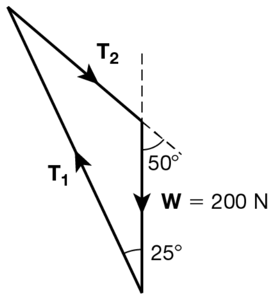
Magnitude of **T**: $T = \frac{W}{\sin 30°}$ (*1 mark*) $= \frac{490.5}{0.5} = 981\text{ N} = 980\text{ N}$ (2 significant figures) (*1 mark*)

7 a



(*1 mark*)

b The sketch of the triangle of forces (*1 mark*) shows that it has two angles of 25° so it is isosceles and the magnitude of $\mathbf{T_2}$ = magnitude of **W** = 200 N (*1 mark*).



By drawing a perpendicular line and dividing the triangle into two right-angled triangles, T_1, the magnitude of $\mathbf{T_1}$, is given by:

$$\cos 25° = \frac{0.5T_1}{200\,\text{N}}$$

T_1 = (400 N) cos 25° = 363 N = 360 N (2 significant figures) (*1 mark*)

Alternatively solve by scale diagram:

Choose scale, e.g. 1 cm = 40 N.

Draw vertical line 5 cm long to represent **W**.

Draw line from bottom of this line at 25° to represent $\mathbf{T_1}$.

Draw line from top of vertical line at (180° – 50°) = 130° to represent $\mathbf{T_2}$. (*1 mark*)

Measure length of $\mathbf{T_1}$: 9.1 cm, so $\mathbf{T_1}$ = 40 × 9.1 = 364 N = 360 N (2 significant figures). (*1 mark*)

Measure length of $\mathbf{T_2}$: 5 cm, so $\mathbf{T_2}$ = 200 N. (*1 mark*)

c If $\alpha = 50°$ and **BC** stays at 50° to the vertical, this means, looking at the triangle of forces, the direction of $\mathbf{T_1}$ and $\mathbf{T_2}$ are the same – they are parallel and will not meet to form a triangle. As the three forces $\mathbf{T_1}$, $\mathbf{T_2}$, and **W** do not form a triangle of forces, they are not in equilibrium.

Using velocity–time graphs

Specification references

- *3.4.1.3*
- *M3.5 Calculate a rate of change from a graph showing a linear relationship*
- *M3.6 Draw and use the slope of a tangent to a curve as a measure of rate of change*
- *M4.3 Calculate areas of triangles*

Maths Skills for Physics references

- *3.1 Motion 1*
- *3.2 Motion 2*

Learning outcomes

After completing the worksheet you should be able to:

- demonstrate and apply knowledge and understanding of the appearance of a velocity–time graph for an object
- use a velocity–time graph to calculate uniform and non-uniform acceleration
- use a velocity–time graph to calculate the distance travelled in a given time.

Introduction

You should always check carefully whether a graph is a displacement–time graph or a velocity–time graph. Remember the following points for a velocity–time graph:

- A horizontal line means the object is travelling at constant velocity. In Figure 1 in the worked example, the object has a constant velocity of 8 m s^{-1} between points **Q** and **R**.
- The object is stationary when it has a velocity of 0 m s^{-1}. In Figure 1, this happens at points **O** and **S**.
- If the graph has a linear slope, the object is accelerating at a constant rate. In Figure 1, the object has constant acceleration between **O** and **P**, and between **P** and **Q**. The steeper slope between **P** and **Q** shows the acceleration is greater than between **O** and **P**.
- If the slope is non-linear, the acceleration is not constant. In Figure 1, between **R** and **S** the object is slowing down – it has a non-uniform negative acceleration.
- The area under the curve of a velocity–time graph is equal to the distance travelled.
- You can calculate the area under the linear parts of the graph using the formulae for the area of a rectangle and of a triangle, and by counting the squares under a curve.

Worked example 1

Question

Figure 1 shows the motion of an object over 8 s. Use the graph to calculate:

a the acceleration between **P** and **Q**

b the acceleration at time 6.0 s.

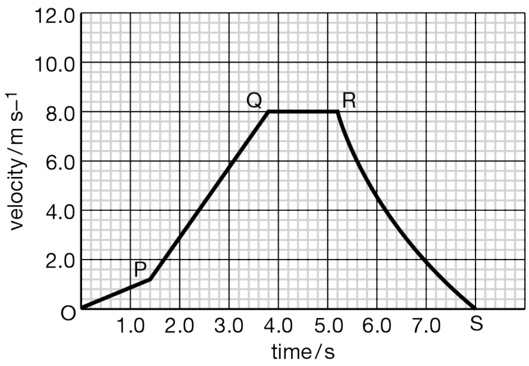


Figure 1

Answer

a *Step 1*

The acceleration between **P** and **Q** can be found from the gradient of the line between points **P** and **Q**.

If the line is very short then you can extend it so the change in velocity and the change in time are large – see Figure 2 where this has been done. This will give a more accurate value. You can use any two points on the straight line that passes through **P** and **Q**. You should make the triangle for calculating a gradient as large as will fit on the graph paper.

Acceleration = gradient of line between **P** and **Q**

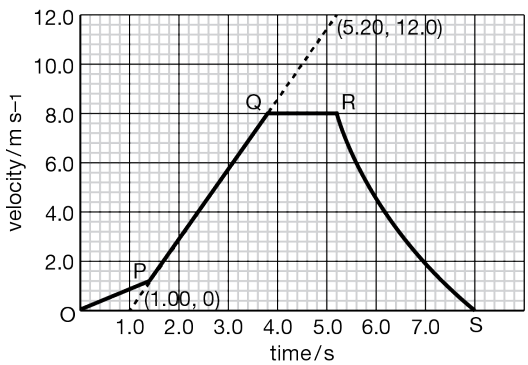


Figure 2

Step 2

Calculate the gradient between points (1.00, 0.0) and (5.20, 12.0).

For a graph of x against y, gradient $= \dfrac{\text{change in } y}{\text{change in } x}$

Gradient $= \dfrac{(12.0 - 0.0)\ \text{m s}^{-1}}{(5.20 - 1.00)\ \text{s}} = 2.86\ \text{m s}^{-2}$

Step 3

Remember to state the acceleration with correct units.

Acceleration between **P** and **Q** = 2.86 m s^{-2}

b *Step 4*

The acceleration between **R** and **S** is not constant. At any time the acceleration is equal to the gradient of the curve at that time. To find the acceleration at time $t = 6.0$ s, you need to draw a tangent to the curve at the point where $t = 6.0$ s and find the gradient of the tangent. The tangent is the line that just touches the curve at that point.

At $t = 6.0$ s, the acceleration = gradient of the tangent when $t = 6.0$ s.

Step 5

Draw the tangent to the curve as shown in Figure 3. Use a transparent ruler and a sharp pencil. Ensure the ruler touches the curve only at the point where the tangent is to be drawn – the ruler should not cross the curve. Extend the tangent far enough to get an accurate value.

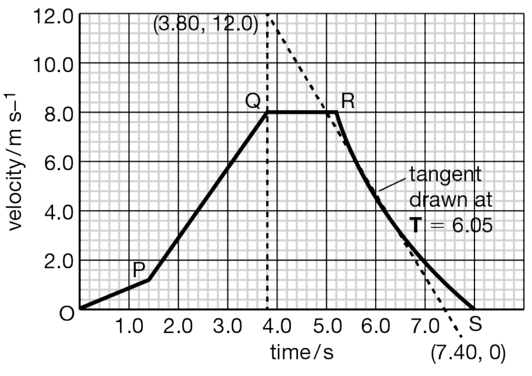


Figure 3

Step 6

Calculate the gradient between points (3.80, 12.0) and (7.40, 0.00). Notice that the gradient is negative because the object is slowing down (decelerating).

Gradient $= \dfrac{(0 - 12.0)\ \text{m s}^{-1}}{(7.40 - 3.80)\ \text{s}} = -3.33\ \text{m s}^{-2}$

Step 7

Remember to state the acceleration with correct units and to say that it is negative. Alternatively, state it as a deceleration and leave the value positive.

Acceleration at $t = 6.0$ s = -3.33 m s^{-2}

Deceleration at $t = 6.0$ s = 3.33 m s^{-2}

Question

1 For the velocity–time graph in Figure 1:

- **a** state the velocity after the object has been moving for 3.4 s (*1 mark*)
- **b** state the velocity after the object has been moving for 5.6 s (*1 mark*)
- **c** calculate the acceleration between **O** and **P** (*1 mark*)
- **d** calculate the acceleration at $t = 7.0$ s. (*1 mark*)

Worked example 2

Question

Figure 4 shows the motion of a toy car. Calculate how far it has it travelled after 5.2 s.

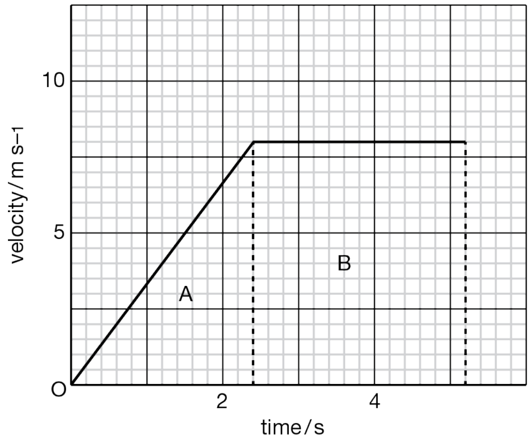


Figure 4

Answer

Step 1

The distance travelled is represented by the area under the graph. Separate the area into rectangles and triangles. If you make a mistake later on, then this step may still be worth some marks.

Distance travelled = area under curve = area A + area B

Step 2

Calculate the area of triangle A using the formula:

area of triangle = $\frac{1}{2}$ base × perpendicular height.

Notice that this area represents distance, which has the unit metre. See '2 Calculation sheet: Using S.I. units' for more information on units.

Base = (2.4 – 0) m s^{-1} = 2.4 m s^{-1}

Height = (8.0 – 0) m s^{-1} = 8.0 m s^{-1}

Area of A = 0.5 × 2.40 s × 8.0 m s^{-1}

Area A = 9.6 m

Step 3

Calculate the area of rectangle B using the formula:
area of rectangle = base × height.

Base = (5.2 – 2.4) m s^{-1} = 2.8 m s^{-1}

Height = (8.0 – 0) m s^{-1} = 8.0 m s^{-1}

Area of B = 2.80 s × 8.0 m s^{-1}

Area B = 22.4 m

Step 4

Add the distances to get the total.

Distance travelled = 9.6 m + 22.4 m

Distance travelled = 32.0 m

Question

2 Calculate the distance travelled during the journey shown in Figure 5. (*2 marks*)

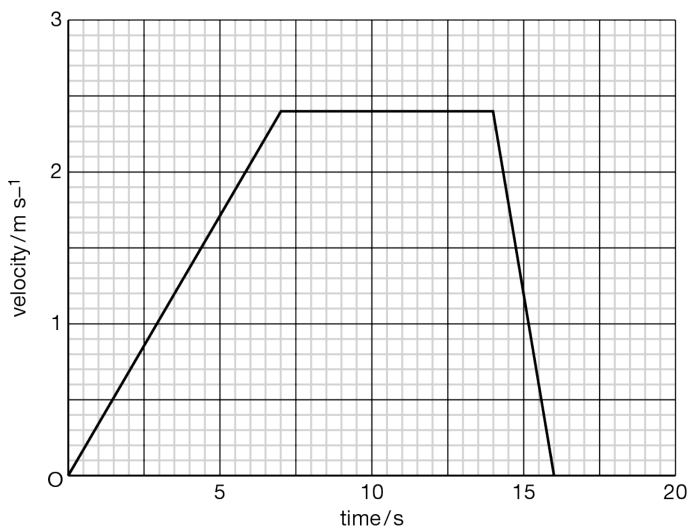


Figure 5

Worked example 3

Question

Calculate the distance travelled in the car journey shown in Figure 6.

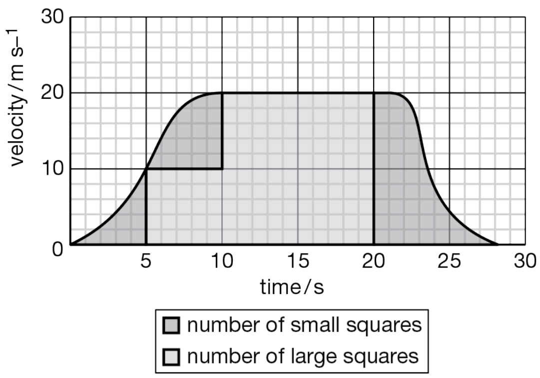


Figure 6

Answer

Step 1

Write down that the distance travelled is equal to the area under the curve:

Distance travelled = area under the curve

Step 2

Count the complete large squares under the graph, crossing them off as you go.

Number of large squares = 5

Step 3

Count the remaining complete small squares under the graph, crossing them off as you go. Count all the squares of which are half or more under the curve, leave out those where less than half is under the curve.

Number of small squares ≈ 69

Step 4

Calculate the distance represented by a small square and a large square. Take care that you have read the scales for the horizontal and vertical axes correctly.

1 small square: area represents $1.0\,\text{s} \times 2.0\,\text{m s}^{-1} = 2.0\,\text{m}$.

1 large square (25 small squares): area represents $5\,\text{s} \times 10\,\text{m s}^{-1} = 50\,\text{m}$.

Step 5

Calculate the area under graph.

Area under graph = (number of large squares × distance represented by one large square) + (number of small squares × distance represented by one small square).

Distance travelled ≈ (5 × 50 m) + (69 × 2.0 m)

≈ 200 m + 138 m

≈ 338 m

≈ 340 m (2 significant figures)

Questions

3 Estimate the distance travelled in Figure 1 on this sheet. (*3 marks*)

4 Figure 7 is a velocity–time graph showing the motion of two cars, P and Q, which are at the same place at $t = 0$ s.

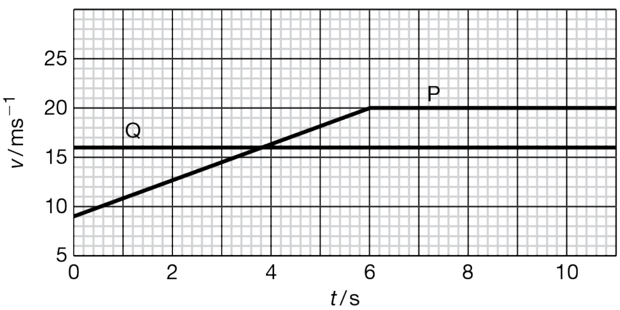


Figure 7

a Describe the motion of P from 0 to 10 s. (*2 marks*)

b Calculate the distance travelled by P in the first 6 s. (*3 marks*)

c Use the graph to identify the time at which both cars have the same velocity. (*1 mark*)

d Determine the time at which car P overtakes car Q. (*4 marks*)

Maths skills links to other areas

You may also need to determine the area under a graph in Topic 11.4 *More about stress and strain*, and Topic 9.2 *Impact forces*.

Answers

1 **a** $6.8\ \text{m s}^{-1}$ (*1 mark*)

b $-6.0\ \text{m s}^{-1}$ (*1 mark*)

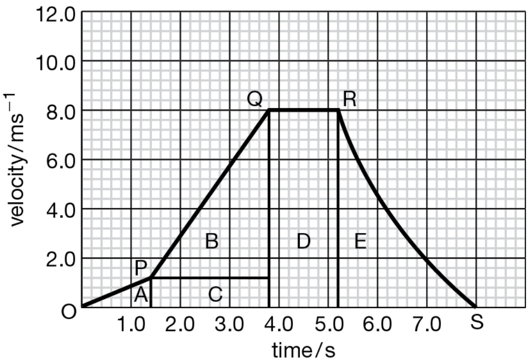
c $\dfrac{(7.6-0.0)\ \text{m s}^{-1}}{(8.2-0.0)\ \text{s}} = 0.9\ \text{m s}^{-2}$ (*1 mark*)

d $\dfrac{(0.0-12.0)\ \text{m s}^{-1}}{(7.8-2.7)\ \text{s}} = -2.4\ \text{m s}^{-2}$ (*1 mark*)

2 Area = $[\frac{1}{2} \times (7.0\ \text{s}) \times (2.4\ \text{m s}^{-1})] + [(14.0\ \text{s} - 7.0\ \text{s}) \times (2.4\ \text{m s}^{-1})] + [\frac{1}{2} \times (2.0\ \text{s}) \times (2.4\ \text{m s}^{-1})]$ (*1 mark*)

Area = 8.4 m +16.8 m + 2.4 m = 27.6 m = 28 m (2 significant figures) (*1 mark*)

3 Award *1 mark* for using the area under the curve, *1 mark* for counting squares or using triangles, and *1 mark* for the correct answer.



One possible method: area = triangle A + triangle B + rectangle C + rectangle D + counting squares under curve E

This gives: $(0.5 \times 1.4\ \text{s} \times 1.2\ \text{m s}^{-1}) + 0.5 \times (3.8 - 1.4)\ \text{s} \times (8.0 - 1.2)\ \text{m s}^{-1} + (3.8 - 1.4)\ \text{s} \times 1.2\ \text{m s}^{-1} + (8.0\ \text{m s}^{-1} \times 1.4\ \text{s}) + 120 \times (0.4\ \text{m s}^{-1} \times 0.2\ \text{s})$

= 0.84 m + 8.16 m + 2.88 m + 9.6 m = 21.48 m = 21 m (2 significant figures)

4 **a** At the start P has a uniform acceleration and accelerates from $9\ \text{m s}^{-1}$ to $20\ \text{m s}^{-1}$ in 6 s. (*1 mark*)

It then continues at a constant speed of $20\ \text{m s}^{-1}$. (*1 mark*)

b Distance travelled in first 6 s = area under graph (*1*

$= (9\ \text{m s}^{-1} \times 6\ \text{s}) + 0.5 \times (20 - 9)\ \text{m s}^{-1} \times 6\ \text{s}$ (*1 mark*)

= 87 m (*1 mark*)

c Both have same *v* when graphs go through same point: after 4.0 s. (*1 mark*)

d P overtakes Q when both have travelled the same distance: after time X where Q has travelled $16X$ and P has travelled [distance in part **b** + $20(X - 6)$ = $84 + 20(X - 6)$].

$16X = 84 + 20(X - 6)$ gives $36 = 4X$ so $X = 9$ s. (*4 marks*)

Elastic and inelastic collisions

Specification references

- *3.4.1.8 Conservation of momentum*
- *3.4.1.6 Momentum*
- *M0.5 Use calculators to find and use power functions*
- *M2.2 Change the subject of an equation, including non-linear equations*
- *M2.3 Substitute numerical values into algebraic equations using appropriate units for physical quantities*
- *M2.4 Solve algebraic equations, including quadratic equations*

Maths Skills for Physics references

- 3.11 Momentum and energy

Learning objectives

After completing the worksheet you should be able to:

- understand and apply the knowledge that in a perfectly elastic collision kinetic energy is conserved
- use the principle of conservation of momentum and the conservation of kinetic energy to solve problems involving perfectly elastic collisions
- understand that both momentum and energy are conserved in all collisions.

Introduction

Two important principles in physics are the conservation of energy and the conservation of momentum.

Energy and momentum are very different quantities:

- momentum is calculated using mass × velocity and is measured in kilogram metres per second (kg m s^{-1}) or newton seconds (N s)
- energy is measured in joules (J) and kinetic energy, for example, is calculated using $\frac{1}{2}$ mass × velocity^2 $\left(\frac{1}{2}mv^2\right)$.

The principle of conservation of energy states that energy is always conserved. (In nuclear reactions, we must take into account the conversion of energy to mass and mass to energy.) When a collision occurs between two objects, the total energy is conserved. But it is difficult to say by how much the thermal energy of the colliding objects and their surroundings increases.

The principle of conservation of momentum states that momentum is always conserved in a collision, as long as no external force is acting on the system. You can use this law to calculate the velocities of colliding objects.

Collisions can be elastic or inelastic. In a perfectly elastic collision, kinetic energy is conserved, whereas in an inelastic collision, kinetic energy is not conserved. To determine if a collision is elastic, you use the principle of conservation of momentum to find the velocities of the objects after the collision, and then calculate the kinetic energy before and after the collision to see if it is conserved. The steps for this are shown below:

- principle of conservation of momentum:
  momentum before collision = momentum after collision
- $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$
- kinetic energy before = $\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$
- kinetic energy after = $\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$
- change in kinetic energy, $\Delta E_k = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 - \left(\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2\right)$
- if the change in kinetic energy $\Delta E_k = 0$, then the collision is elastic.

Almost all real collisions are inelastic. The most extreme inelastic collision occurs when the colliding objects stick together. Perfectly elastic collisions occur between atoms and sub-atomic particles.

Worked example

Question

A neutron collides head-on with a stationary nucleus with a mass twice that of the neutron. The initial velocity of the neutron is 1.2×10^7 m s^{-1}. After the collision it rebounds with velocity 0.40×10^7 m s^{-1}.

Show whether the collision is elastic.

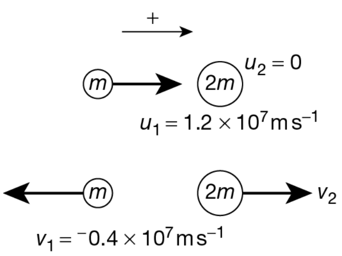
Answer

Step 1: Find the final velocity of the nucleus by using the principle of conservation of momentum. Do not forget to state that you are using this principle.

total momentum before = total momentum after

Step 2: Draw a diagram, showing which direction you are taking as positive, the values of mass and velocity you know, and the one you are calculating.

In this case, name the mass of the neutron *m*, so then the mass of the nucleus is 2*m*.



Step 3: Substitute the values into the equation: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

m_1 = mass of neutron (m)

m_2 = mass of nucleus ($2m$)

u_1 = initial velocity of neutron

u_2 = initial velocity of nucleus

v_1 = final velocity of neutron

v_2 = final velocity of nucleus

$m\,(1.2 \times 10^7\ \text{m s}^{-1}) + 0 = m\,(-0.40 \times 10^7\ \text{m s}^{-1}) + 2m\,v_2$

Step 4: Divide everything by m to remove it from your equation – you can do this as long as m is not zero, and it is not.

$(1.2 \times 10^7) = (-0.40 \times 10^7) + 2v_2$

Step 5: Rearrange the equation to find v_2.

$2v_2 = (1.2 \times 10^7) + (0.40 \times 10^7)$

Step 6: Calculate v_2

$v_2 = \frac{1}{2}\,(1.6 \times 10^7)$

$v_2 = 0.80 \times 10^7\ \text{m s}^{-1}$

Step 7: Use the equation $E_K = \frac{1}{2} m v^2$ to find the kinetic energy before the collision.

Before: $E_K = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$

$E_K = \frac{1}{2} m\,(1.2 \times 10^7)^2$

You can leave this expression for E_K or simplify it further:

$E_K = \frac{1}{2} m\,(1.44 \times 10^{14})$

$E_K = (0.72 \times 10^{14})\,m$

Step 8: Repeat Step 7 for the kinetic energy after the collision.

After: $E_K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

$E_K = \frac{1}{2} m\,(0.40 \times 10^7)^2 + \frac{1}{2}\ 2\,m\,(0.80 \times 10^7)^2$

Step 9: Simplify the equation from Step 8 (kinetic energy after the collision) by taking the $\frac{1}{2}m$ outside the brackets. Take care with the squared numbers.

$E_K = \frac{1}{2} m\,[0.40^2 + 2 \times 0.80^2] \times (10^7)^2$

Step 10: Simplify the calculation inside the square brackets. You are aiming to get the same expression as for the E_K before the collision (which you found in Step 7) – this will only work if E_K is conserved.

$E_K = \frac{1}{2} m\,(0.16 \times 10^7 + 2 \times 0.64 \times 10^7)^2$

$E_K = \frac{1}{2} m\,[0.16 + 2 \times 0.64] \times (10^7)^2$

$E_K = \frac{1}{2} m\,[1.44] \times (10^{14})$

$E_K = (0.72 \times 10^{14})\,m$

or

$E_K = \frac{1}{2} m\,(1.2 \times 10^7)^2$ (if this is how you left the expression in Step 7)

Step 11: Remember to answer the question by stating whether the collision is elastic, and how you know.

kinetic energy before the collision = kinetic energy after the collision

Therefore, the collision is perfectly elastic.

Questions

1 Two snooker balls, A and B, with the same mass move towards each other and collide. The initial velocity for A is $+0.3\ \mathrm{m\ s^{-1}}$, for and B is $-0.2\ \mathrm{m\ s^{-1}}$. The final velocity of A is $-0.2\ \mathrm{m\ s^{-1}}$.

a Determine the final velocity of B. (*1 mark*)

b Show whether the collision is elastic. (*2 marks*)

2 A mass of 5.00 kg moving with velocity $20.0\ \mathrm{m\ s^{-1}}$ to the right collides with a stationary mass of 10.0 kg. The final velocity of the 5.00 kg mass is $6.67\ \mathrm{m\ s^{-1}}$ to the left.

a Calculate the final velocity of the 10.0 kg mass. (*2 marks*)

b Is the collision elastic? (*2 marks*)

3 A 1.0 kg mass with initial velocity $5.0\ \mathrm{m\ s^{-1}}$ collides with, and sticks to, a stationary 6.0 kg mass. The combined mass collides with, and sticks to, a stationary 3.0 kg mass. The collisions are all head-on.

Calculate:

a the final velocity (*2 marks*)

b the kinetic energy lost. (*3 marks*)

4 An alpha particle of mass 4.0 u with a velocity of $1.0 \times 10^6\ \mathrm{m\ s^{-1}}$ to the right collides with a stationary proton of mass 1.0 u. After the collision, the alpha particle moves with velocity $0.60 \times 10^6\ \mathrm{m\ s^{-1}}$ to the right.

a Calculate the velocity of the proton. (*2 marks*)

b Show that the collision is elastic. (*3 marks*)

Maths skills links to other areas

You may need to use similar calculations, and apply the principle of the conservation of energy and the conservation of momentum in other areas of the course. For example, in Topic 9.3 *Conservation of momentum* and Topic 10.1 *Work and energy.*

Answers

1 **a** By conservation of momentum: $m\,(0.3\ \text{m s}^{-1}) + m\,(-0.2\ \text{m s}^{-1}) = m\,(-0.2\ \text{m s}^{-1}) + mv_2$

$v_2 = +0.3\ \text{m s}^{-1}$ (*1 mark*)

b Use $\frac{1}{2}mv^2$ to show total KE before = total KE after.

Before: $\frac{1}{2}m\,(0.3\ \text{m s}^{-1})^2 + \frac{1}{2}m\,(-0.2\ \text{m s}^{-1})^2$ (*1 mark*)

After: $\frac{1}{2}m\,(-0.2\ \text{m s}^{-1})^2 + \frac{1}{2}m\,(0.3\ \text{m s}^{-1})^2$

These are equal therefore the collision is elastic.(*1 mark* for KE after correct calculation and correct st

2 **a** $(5.0\ \text{kg})\,(20.0\ \text{m s}^{-1}) + (0) = (5.0\ \text{kg})\,(-6.67\ \text{m s}^{-1}) + (10.0\ \text{kg})\,v_2$ (*1 mark*)

$v_2 = 13.35\ \text{m s}^{-1} = 13.3\ \text{m s}^{-1}$ (three significant figures) (*1 mark*)

b E_K before $= \frac{1}{2}\,(5.0\ \text{kg})\,(20.0\ \text{m s}^{-1})^2 = 1000\ \text{J}$

E_K after $= \frac{1}{2}\,(5.0\ \text{kg})\,(6.67\ \text{m s}^{-1})^2 + \frac{1}{2}\,(10.0\ \text{kg})\,(13.35\ \text{m s}^{-1})^2 = 1002\ \text{J} = 1000\ \text{J}$
(three significant figures)

Therefore, the collision is elastic to three significant figures.

Award *1 mark* for correct calculation of KE before **or** after, award *2 marks* for correct calculation of KE before **and** after, and correct statement.

3 **a** By conservation of momentum: $(1.0\ \text{kg})\,(5.0\ \text{m s}^{-1}) = (7.0\ \text{kg})\,v_1$

$v_1 = \frac{5.0}{7.0}\ \text{m s}^{-1}$ (*1 mark*)

By conservation of momentum: $(7.0\ \text{kg})\,\frac{5.0}{7.0}\ \text{m s}^{-1} = (10.0\ \text{kg})\,v_2$

$v_2 = \frac{5.0}{10.0}\ \text{m s}^{-1} = 0.5\ \text{m s}^{-1}$ (*1 mark*)

b E_K before: $\frac{1}{2}\,(1.0\ \text{kg})\,(5.0\ \text{m s}^{-1})^2 = 12.5\ \text{J}$ (*1 mark*)

E_K after: $\frac{1}{2}\,(10.0\ \text{kg})\,(0.50\ \text{m s}^{-1})^2 = 1.25\ \text{J}$ (*1 mark*)

E_K lost: $12.5\ \text{J} - 1.25\ \text{J} = 11.25\ \text{J}$ (*1 mark*)

4 **a** By conservation of momentum:

$(4\text{u})\,(1.0 \times 10^6\ \text{m s}^{-1}) + (0) = (4\text{u})\,(0.60 \times 10^6\ \text{m s}^{-1}) + (1\text{u})\,v_2$ (*1 mark*)

$v_2 = 1.6 \times 10^6\ \text{m s}^{-1}$ (*1 mark*)

b E_K before $= \frac{1}{2}\,(4\text{u})\,(1.0 \times 10^6\ \text{m s}^{-})^2$ (*1 mark*)

E_K after = $\frac{1}{2}$ (4u)(0.60 × 10^6 m $s^{-1})^2$ + $\frac{1}{2}$ (1u) (1.6 × 10^6 m $s^{-1})^2$ = $\frac{1}{2}$ (4u)(1.0 × 10^6 m $s^{-})^2$ (*1 mark*)

E_K before = E_K after, therefore the collision is elastic. (*1 mark*)

Students may try to convert from u to kg – you may wish to explain that this is unnecessary in this case, but if done correctly they would still gain full marks.

Understanding Hooke's law

Specification references

- *3.4.2.1 Bulk properties of solids*
- *M2.1 Understands and use the symbols: =, ∝*
- *M3.2 Plot two variables from experimental or other data*
- *M3.4 Determine the slope and intercept of a linear graph*
- *M3.1 Translate information between graphical, numerical and algebraic forms*

Maths skills for Physics references

- *4.1 Elasticity 1*

Learning objectives

After completing the worksheet you should be able to:

- demonstrate knowledge and understanding of Hooke's law
- plot and interpret graphs of force against extension (or compression) for springs and wires
- apply knowledge and understanding of extension and compression to springs, wires, and samples of other materials.

Introduction

If you stretch a solid it is in *tension*. The forces applied are tensile forces and there is an *extension* – an increase in length.

If you compress a solid it is in *compression*. The forces applied are compressive forces and there is a *decrease in length*.

When a material obeys Hooke's law the extension, ΔL, is directly proportional to the force applied, F, as long as the limit of proportionality is not exceeded.

$F \propto L$ or $F = k\,\Delta L$, where k is the stiffness constant for the wire or spring. Units of k are $\mathrm{N\,m^{-1}}$.

$$k = \frac{F}{x}$$

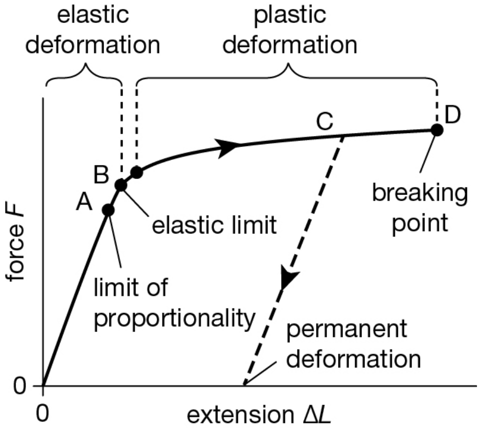


Figure 1

Figure 1 is a force–extension graph for a metal. OA is the region where Hooke's law is obeyed. Other metals, and other materials, will have different graphs depending on their properties. Notice the following two points.

- The elastic limit (B) is the point after which the material no longer returns to its original length. This is not always the same as the limit of proportionality (A).
- When a material starts to experience plastic deformation, there is a much bigger *increase* in extension for an *increase* in force. (It is always true that there is a bigger extension for a bigger force, so be careful how you explain the change.)

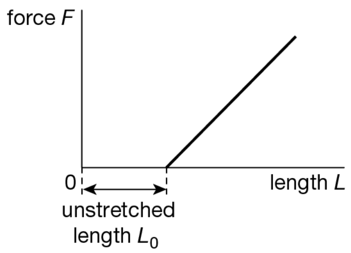


Figure 2

Figure 2 is a force–length graph for the same metal as Figure 1. Notice that whilst the force is directly proportional to extension (see Figure 1) it is incorrect to say that force is directly proportional to the length, L, of the wire. A graph of F against ΔL goes through the origin (0,0), but a graph of F against L goes through the point where
$F = 0$ and L is the unstretched length (or natural length) of the wire, L_0. This is a linear relationship: $F = k(L - L_0)$.

Worked example

Question

Figure 3 is a graph of force against extension for a wire.

a Deduce the applied force and the length of the wire when it reaches the limit of proportionality.

b Calculate the stiffness constant for the wire.

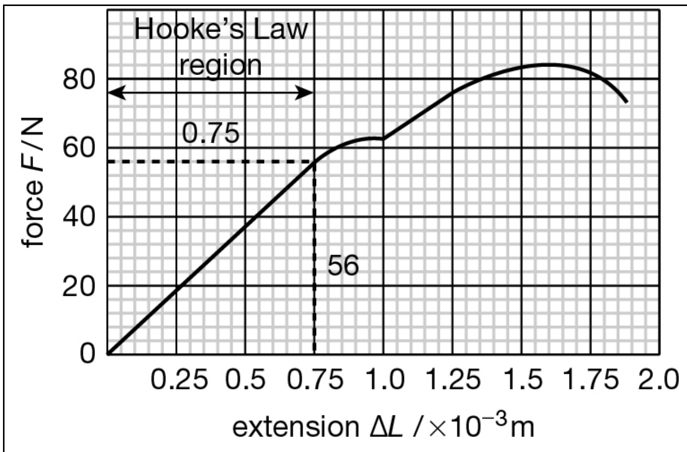


Figure 3

Answer

a *Step 1*

Mark, on the graph, the point at the end of the straight-line section of the graph.

Step 2

Use a transparent ruler to line the point up with the force axis and read off the force, taking care with the scale of the graph. Note down the answer.

Force, $F = 56\ldots$

Step 3

Check the axis for the unit and any scale factor, and multiply your answer by these.

Force, $F = 56\,\text{N}$

Step 4

Repeat Steps 1 to 3 for the extension axis.

Extension, $\Delta L = 0.75...$

Extension, $\Delta L = 0.75 \times 10^{-3}\,\text{m}$

b *Step 5*

State that the stiffness constant is equal to the gradient of the graph, and show the gradient is the change in force divided by the change in extension over a long straight part of the graph.

The stiffness constant, *k*, is equal to the gradient of the graph:

$$k = \frac{56\,\text{N}}{0.75 \times 10^{-3}\,\text{m}}.$$

Step 6

Use you calculator to calculate the value, and do not forget the units.

$k = 75\,000\,\text{N m}^{-1}$ (two significant figures)

Questions

1 A force of 160 N extends a copper wire by 2.7 cm. Calculate the stiffness constant of the wire. (*1 mark*)

2 A 12 N weight is hung on a spring 5.0 cm long with spring constant 8500 N m^{-1}. Calculate:

a the extension (*1 mark*)

b the new length of the spring. (*1 mark*)

3 A horizontal plank is supported at each end. A child of mass 55 kg stands on the centre of the plank, which bends, so the centre moves down 4.0 mm.

Calculate the stiffness constant of the plank. (*1 mark*)

4 A student has two identical springs with spring constant 240 N m^{-1} and natural length 210 mm. The weight of the springs is negligible. Calculate the length of each of the springs when:

a they are joined vertically and stretched with a weight of 8.0 N (*2 marks*)

b they are joined in parallel and stretched with a weight of 8.0 N. (*2 marks*)

5 Force is applied to a spring and this data is collected:

Force / N	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0
Length / cm	2.6	2.8	3.0	3.2	3.4	3.6	3.9	4.3

a Plot a force–extension graph for the spring. (*5 marks*)

b Explain the shape of the graph. (*3 marks*)

c Calculate the spring constant. (*2 marks*)

d Suggest and explain your choice for:

i a material that would give a similarly shaped graph (*1 mark*)

ii a material that would not give a similarly shaped graph. (*2 marks*)

Maths skills links to other areas

You will need to be able to plot and interpret graphs throughout the AS Level course, both from data provided and from your own experimental results; for example, when investigating distance–time graphs in Topic 7.1 *Speed and velocity*.

Answers

6 $k = \frac{F}{\Delta L} = \frac{160\ \text{N}}{2.7 \times 10^{-2}\ \text{m}} = 5900\ \text{N m}^{-1}$ (*1 mark*)

7 **a** $\Delta L = \frac{F}{k} = 1\frac{12\ \text{N}}{8500\ \text{N m}^{-1}} = 1.41 \times 10^{-3}\ \text{m} = 1.4\ \text{mm}$ (*1 mark*)

b $L = 5.0\ \text{cm} + 1.41\ \text{mm} = 5.141\ \text{cm} = 5.1\ \text{cm}$ (two significant figures) (*1 mark*)

8 $F = (55\ \text{kg}) \times (9.81\ \text{m s}^{-2})$; $\Delta L = 4.0\ \text{mm}$

$k = \frac{(155\text{kg}) \times (9.81\text{ms}^{-2})}{(4.0 \times 10^{-3}\text{m})} = 1.3 \times 10^5\ \text{N m}^{-1}$ (*1 mark*)

9 **a** Vertically the force on both springs is 8.0 N (imagine each spring in turn removed or replaced with an inextensible wire). For each spring:

$\Delta L = \frac{F}{k} = \frac{8.0\ \text{N}}{240\ \text{N m}^{-1}}$

$= 0.0333\ \text{m} = 33\ \text{mm}$ (two significant figures) (*1 mark*)

Length of each spring $= 210\ \text{mm} + 33\ \text{mm}$

$= 243\ \text{mm} = 240\ \text{mm}$ (two significant figures) (*1 mark*)

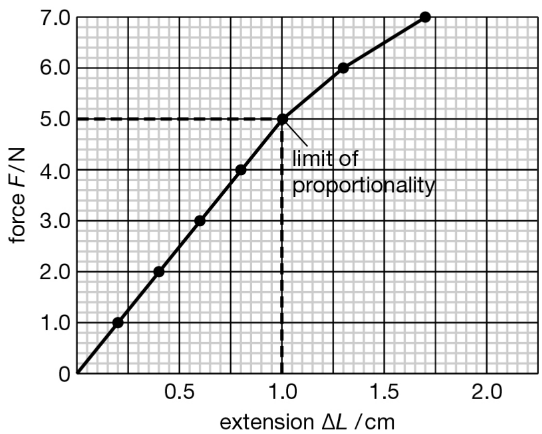
b Connected in parallel the two springs take the weight, so the force on each spring $= \frac{F}{2}$

$= 4.0\ \text{N}$

$\Delta L = \frac{F}{k} = \frac{4.0\ \text{N}}{240\ \text{N m}^{-1}} = 0.0167\ \text{m} = 17\ \text{mm}$ (two significant figures) (*1 mark*)

Length of each spring $= 210\ \text{mm} + 17\ \text{mm} = 227\ \text{mm} = 230\ \text{mm}$ (two significant figures) (*1 mark*)

10 **a**



Note that extensions are found by subtracting the length at $F = 0$ (2.6 cm) from all the lengths.

Award up to *5 marks* from the following points.

- Force on vertical axis labelled force **or** F; extension on horizontal axis labelled extension **or** ΔL. (*1 mark*)
- Units of /N **and** /cm or / $\times 10^{-2}$ m shown on axes. (*1 mark*)
- Extensions correctly calculated. (*1 mark*)
- Points correctly plotted to within half a small square (e.c.f. calculated extensions). (*1 mark*)
- Straight line drawn through points and origin where these are straight, then curves as shown. (*1 mark*)

b The answer should include the following points.

- The limit of proportionality is marked on the graph (or state coordinates (5.0, 1.0)). (*1 mark*)
- Force is directly proportional to extension from 0 to 5 N so graph goes through origin and is a straight line. In this region the spring obeys Hooke's law. (*1 mark*)
- The limit of proportionality is reached at, or just beyond, 5 N as the graph then curves. An increase in force then produces a greater increase in extension since the material is undergoing plastic deformation. (*1 mark*)

c Students should calculate the gradient of the graph before the limit of proportionality is reached. (*1 mark*)

$$k = \frac{F}{\Delta L} = \frac{5.0\,\text{N}}{1.0 \times 10^{-2}\,\text{m}} = 500\,\text{Nm}^{-1}$$ (*1 mark*)

d **i** Allow any correct answer. For example, a metal such as copper would show this behaviour. (Also, brass, aluminium, or cast iron, but not steel, which has a yield point, glass, which is brittle, or rubber, which does not obey Hooke's law.) Copper obeys Hooke's law up to a certain force and then it is ductile and has a region where it deforms plastically. (*1 mark*)

ii Allow any correct answer. For example, glass is brittle. It obeys Hooke's law up to a certain force, but then it snaps without any plastic deformation. (*1 mark* for a material and *1 mark* for an explanation)

Elastic energy

Specification references

- *3.4.2.1 Bulk properties of solids*
- *M0.5 Use calculators to find and use power functions*
- *M3.1 Translate information between graphical, numerical, and algebraic forms*
- *M3.8 Understand the possible physical significance of the area between a curve and the x-axis and be able to calculate it, or estimate it by graphical methods as appropriate*
- *M3.12 Sketch relationships which are modelled by* $y = k\Delta L$, *and* $y = k\Delta L^2$

Maths Skills for Physics references

- *4.1 Elasticity 1*

Learning objectives

After completing the worksheet you should be able to:

- understand that the work done in stretching (or compressing) a material is equal to the area under a force–extension (or compression) graph
- deduce the work done in stretching (or compressing) a material from a graph of force–extension (or compression)
- calculate elastic potential energy when a material is stretched or compressed, using $E = \frac{1}{2}F\Delta L$ or $E = \frac{1}{2}k\Delta L^2$

Introduction

When you stretch or compress a material, work is done by the force you exert on the material. If the material is elastic then it gains elastic potential energy, E, which is released when it returns to its original shape. When plastic deformation occurs, the force deforms the material permanently and energy is transferred by heating.

The work done by any force can be found using a force–distance graph. When a constant force, F, acts over a distance, ΔL, the work done can be calculated from the formula $W = F\Delta L$. On a force–distance graph, this would be the area of the rectangle between the horizontal line and the x-axis (see Figure 1a). If the force is changing, the work done is the area between the line or curve of the graph and the x-axis (see Figure 1b).

In the case of a force *stretching* a material, as the stretching force (or the tension) increases the extension increases.

In the linear region of a force–extension graph, the area under the line can be calculated using the formula for the area of a triangle.

The elastic potential energy, E, equals the work done on the material by the stretching force.

$E = \frac{1}{2} F \Delta L$ and $F = k \Delta L^2$

Therefore, $E = \frac{1}{2} k \Delta L^2$

Out of the linear region, the work done can be found by counting the squares under the curve.

Worked example

Question

Figure 1a shows the force–extension graph for a material that stretches by obeying Hooke's law but does not go back to its original length.

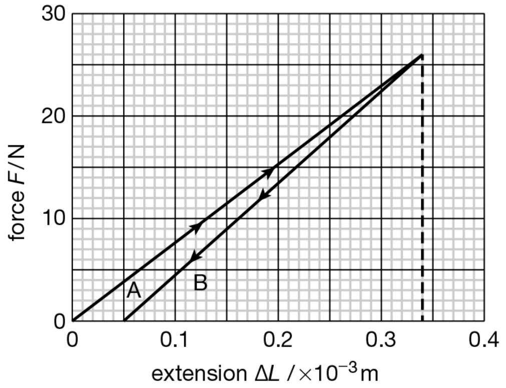


Figure 1a

Calculate:

a the work done in stretching the wire

b the energy recovered when the force is removed

c the energy dissipated in heating the wire as the wire is stretched by the force.

Answer

a *Step 1*

Identify the area that represents the work done on the wire. This is the area of the large triangle, made up of triangles A and B.

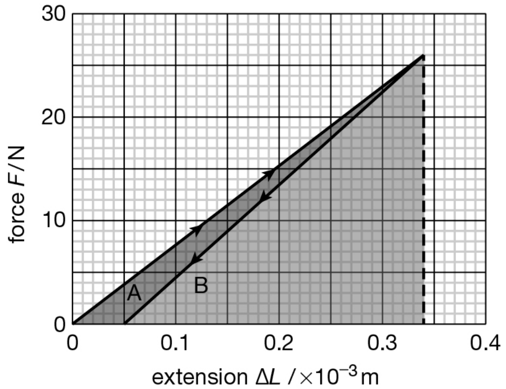


Figure 1b

Work done loading (stretching): W_L = area of triangle (A + B)

Step 2

Use the formula: triangle area = $\frac{1}{2}$ base × height, and substitute values for the base and perpendicular height.

$W_L = \frac{1}{2}\,(0.34 \times 10^{-3}\,\text{m}) \times (26\,\text{N})$

Step 3

State the answer, with units, to two significant figures.

$W_L = 4.42 \times 10^{-3}\,\text{J}$

$= 4.4 \times 10^{-3}\,\text{J}$ (two significant figures)

b *Step 4*

Identify the area that represents the energy recovered by the wire when the force is removed.

The area of triangle B represents the energy recovered by the wire when the force is removed. It is the potential energy stored in the wire when it is stretched. This energy is recovered when the force on the wire is removed and it returns to its original length. This is the work done *by* the wire as it is unloaded and its extension decreases.

The potential energy, E_P, equals the energy transferred when the wire is unloaded, which equals the area of triangle B.

Step 5

Use the formula: triangle area = $\frac{1}{2}$ base × height, and substitute values for the base and perpendicular height.

$E_P = \frac{1}{2}\,[(0.34 \times 10^{-3}\,\text{m}) - (0.05 \times 10^{-3}\,\text{m})] \times (26\,\text{N})$

Step 6

State the answer, with units, to two significant figures.

$E_P = 3.77 \times 10^{-3}\,\text{J}$

$= 3.8 \times 10^{-3}\,\text{J}$ (two significant figures)

c *Step 7*

Explain that the energy dissipated in heating the wire is the difference between the work done by the force stretching the wire and the potential energy stored in the stretched wire (the energy recovered in unloading).

Energy transferred as heat: $W_L - E_P = 4.42 \times 10^{-3}\,\text{J} - 3.77 \times 10^{-3}\,\text{J}$

Step 8

State the answer, with units, to two significant figures.

Energy transferred as heat: 6.5×10^{-4} J (two significant figures)

Questions

1 When a wire is stretched by a weight of 8.0 N it extends by 1.2 mm. The wire obeys Hooke's law and when the load is removed it reverts back to its original length.

Calculate the potential energy stored in the wire when the extension is 1.2 mm. (*1 mark*)

2 A wire has a force constant of 7000 N m^{-1}. The wire stretches 3.0 mm within the linear region of the graph.

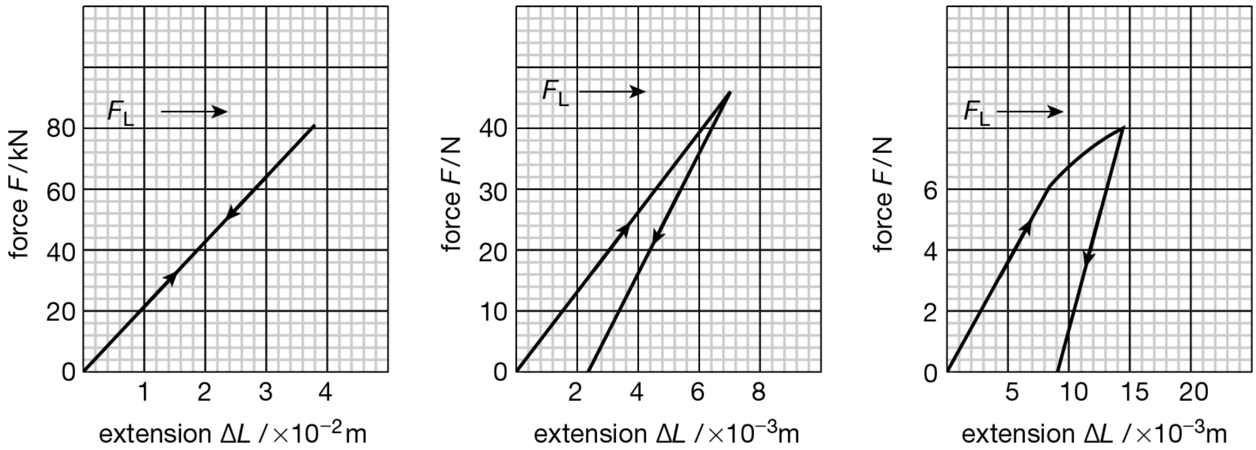
Calculate the elastic potential energy stored in the wire when it is stretched by 3.0 mm. (*1 mark*)

3 Figure 2 shows three graphs of a wire being stretched with a force F_L (different in each case). For each wire, **a**, **b**, and **c**, calculate:

i the work done by the stretching force, F_L (*1 mark for each wire*)

ii the potential energy stored (*1 mark for each wire*)

iii the energy dissipated in heating the wire and its surroundings when the wire is loaded with F_L. (*1 mark for each wire*)



(a) (b) (c)

Figure 2

Hint: For graph **c** you will need to break the area up into rectangles and triangles.

Worked example

Question

Figure 3 shows a force–extension graph for a rubber band when masses were added one by one to a hanger suspended from the rubber band, and then removed one by one.

Calculate the energy dissipated in heating the rubber band during the loading.

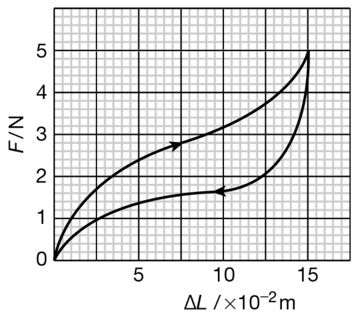


Figure 3

Answer

Step 1

Explain that the energy dissipated in heating is represented by the area between the loading graph and the unloading graph.

The energy dissipated in heating equals the area between the loading and unloading lines on the force–extension graph.

Step 2

Count the squares – including all those that are half a square or more, and omitting all those less than half a square.

(Try to be as accurate as you can. There will be a range of acceptable answers that are ± a few small squares. It is a good idea to make a small mark in each square as you count it to help you to keep count.)

There are 168 small squares.

Step 3

Explain how to convert this answer to energy using the scale on each axis.

Each small square $= (0.20\ \text{N}) \times (0.50 \times 10^{-2}\ \text{m})$

$= 1 \times 10^{-3}\ \text{J}$

Step 4

Calculate the energy and state the answer to two significant figures.

$E = 168 \times 1 \times 10^{-3}\ \text{J}$

$= 0.168\ \text{J}$

$= 0.17\ \text{J}$ (two significant figures)

Questions

4 A piece of elastic 12.0 cm long that is stretched to a length of 23.6 cm and released produces the force–extension graph shown in Figure 4. Use the graph to deduce the energy dissipated in heating the band when the band is stretched to this length, and then released. (*2 marks*)

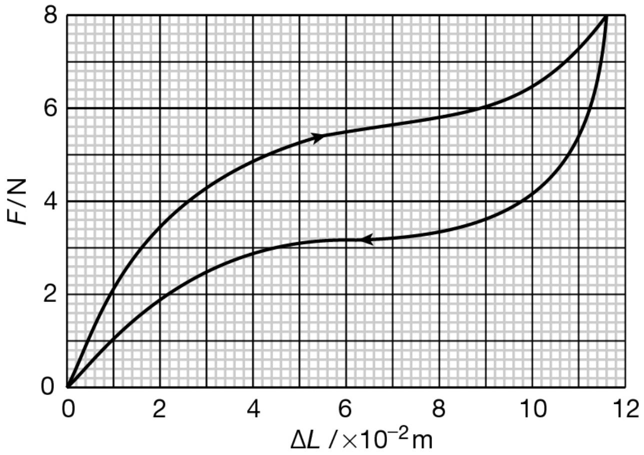


Figure 4

5 The graph in Figure 5 shows the extension of a spring when a tensile force is applied.

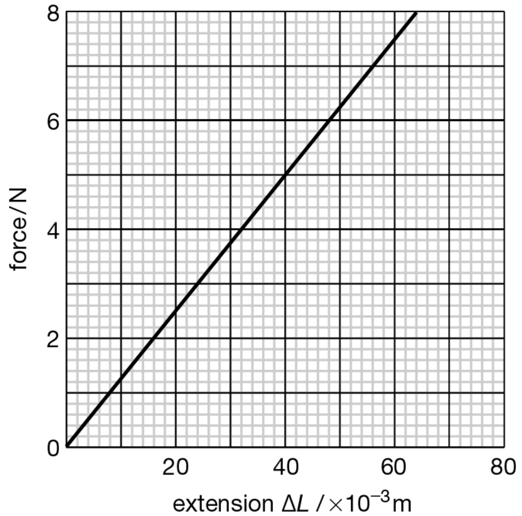


Figure 5

a Use the graph to determine:

i the force constant of the spring (*2 marks*)

ii the work done when a tensile force of 4.0 N is applied to the unstretched spring. (*1 mark*)

b **i** Calculate the potential energy stored in the spring when the unstretched spring is stretched by 82 mm. (*1 mark*)

ii Write down one assumption that you made in calculating your answer to **i**. (*1 mark*)

c The energy stored in the spring is used to launch a small sphere of mass 10 g horizontally.

Assuming all the energy is transferred to the sphere, calculate its initial velocity. (*2 marks*)

Maths skills links to other areas

You will need to be able to plot and interpret graphs throughout the AS Level course, both from data provided and from your own experimental results; for example, when investigating *I–V* characteristics of ohmic and non-ohmic components in Topic 12.4 *Components and their characteristics*. You will also be expected to understand the possible physical significance of the gradient and area under a graph in the graphs you plot.

Answers

1 $E_P = \frac{1}{2} \Delta L$

$= \frac{1}{2}$ (8.0 N) × (1.2 × 10^{-3} m)

$= 4.8 \times 10^{-3}$ J (*1 mark*)

2 $E_P = \frac{1}{2} \Delta L^2 = \frac{1}{2}$ (7000 N m^{-1}) × (3.0 × 10^{-3} m)^2 = 0.0315 J = 0.032 J (two significant figures) (*1 mark*)

3 **a** **i** W = area of triangle with base (0 to 3.8 × 10^{-2} m) and height F_L = (80 kN)

$W = \frac{1}{2}$ (80 kN) × (3.8 × 10^{-2} m) = 1520 J (*1 mark*)

ii All this energy is transferred as potential energy, because it is all recovered when the wire is unloaded. E_P = 1520 J (*1 mark*)

iii Energy transferred as heat = 0 because all energy is transferred as potential energy (*1 mark*)

b **i** W = area of triangle with base (0 to 7.2 × 10^{-3} m) and height F_L = (46.0 N)

$W = \frac{1}{2}$ (46.0 N) × (7.2 × 10^{-3} m) = 0.1656 J = 0.17 J (two significant figures) (*1 mark*)

ii E_P = work done under unloading curve. This is what was stored and is now released.

W = area of triangle with base (2.4 × 10^{-3} m to 7.2 × 10^{-3} m) and height F_L = (46.0 N)

$W = \frac{1}{2}$ (46.0 N) × [(7.2 – 2.4) × 10^{-3} m] = 0.1104 J = 0.11 J (two significant figures) (*1 mark*)

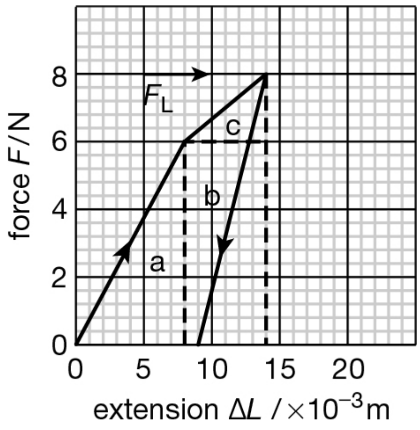
iii Energy transferred as heat = difference between **i** and **ii** = 0.1656 – 0.1104 = 0.0552 J = 0.055 J

or use the area of the triangle formed by the unloading and loading curves:

$E = \frac{1}{2}$ (46.0 N) × (2.4 × 10^{-3} m) = 0.0552 J = 0.055 J (two significant figures) (*1 mark*)

c **i** W = area under curve with base (0 to 14.0 × 10^{-3} m) and height F_L = (8.0 N)

Divide it into areas a, b, and c (a shape can sometimes be approximated to a triangle). You can then use the formulae for areas of rectangles and triangles (or trapeziums if you prefer).



W = area a + area b + area c

$W = \frac{1}{2}$ (6.0 N) × (8.0 × 10^{-3} m) + (6.0 N) × [(14.0 – 8.0) × 10^{-3} m] + $\frac{1}{2}$ (8.0 N) [(14.0 – 8.0) × 10^{-3} m]

= 24.0 × 10^{-3} J + 36.0 × 10^{-3} J + 24 × 10^{-3} J = 84.0 × 10^{-3} J

= 8.4 × 10^{-2} J (two significant figures) (*1 mark*)

ii E_P = work done under unloading curve. This is what was stored and is now released.

W = area under curve with base (9.0 to 14.0 × 10^{-3} m) and height F_L = (8.0 N)

$W = \frac{1}{2}$ (8.0 N) × [(14.0 – 9.0) × 10^{-3} m]

= 20.0 × 10^{-3} J

= 2.0 × 10^{-2} J (two significant figures) (*1 mark*)

iii Energy transferred as heat = difference between **i** and **ii**

= 8.4 × 10^{-2} J – 2.0 × 10^{-2} J = 6.4 × 10^{-2} J (two significant figures) (*1 mark*)

4 This question gives lengths rather than extensions. The extension is 23.6 cm – 12 cm = 11.6 cm. Looking at the graph you can see that it is a graph of a material stretched to an extension of 11.6 cm and released. The energy transferred as heat is the energy inside the shape formed by the loading and unloading curves.

By counting squares, there are 140 ± 5 small squares. (*1 mark*)

Each square represents (0.40 N) × (0.40 × 10^{-2} m) = 1.60 × 10^{-3} J

E = 140 × 1.60 × 10^{-3} J = 0.224 J (*1 mark*)

(135 squares gives 0.216 J and 145 squares gives 0.232 J so your answer should be in this range.)

5 a i k = gradient of graph

$$= \frac{(8.00)\,\text{N}}{(64-0)\times 10^{-3}\,\text{m}}$$ (*1 mark*)

= 125 Nm^{-1} = 130 N m^{-1} (two significant figures) (*1 mark*)

ii When F = 4.0 N, x = 38 × 10^{-3} m

$$W = \frac{1}{2} F \, \Delta L$$

$$= \frac{1}{2} (4.0\,\text{N}) \times (32 \times 10^{-3}\,\text{m})$$

$$= 0.064\,\text{J}$$

(*1 mark*)

b **i** $E_P = \frac{1}{2} k \, \Delta L^2$

$$= \frac{1}{2} (125\,\text{N m}^{-1}) \times (82 \times 10^{-3}\,\text{m})^2$$

$= 0.420\,\text{J} = 0.42\,\text{J}$ (two significant figures) (*1 mark*)

ii The limit of proportionality has not been exceeded **or** the extension is still proportional to the applied force when $\Delta L = 82$ mm. (*1 mark*)

c $E_P = 0.420$ J all transferred to 10 g sphere ($= 10 \times 10^{-3}$ kg), which then has $E_K = \frac{1}{2} m v^2$

$\frac{1}{2} m v^2 = 0.420\,\text{J}$ (*1 mark*)

$v = \sqrt{\frac{2 \times 0.420}{10 \times 10^{-3}\,\text{kg}}} = 9.2\,\text{m s}^{-1}$ (*1 mark*)