HAMPSTEAD SCHOOL

Learning together Achieving together

YR11-12

Summer Bridging Tasks

2023

Further Maths A Level

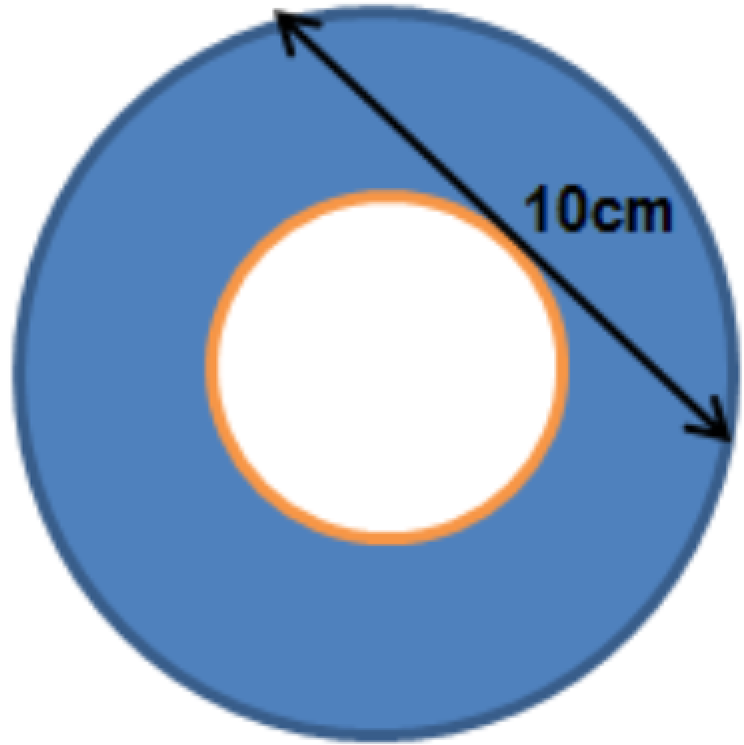
Name: ____

- You should spend some time during the summer holidays working on the activities in this booklet.
- You will be required to hand in this booklet in your first lesson at the start of Year 12 and the content will be used to form the basis of your first assessments.
- You should try your best and show commitment to your studies.

- We are really looking forward to you coming to Hampstead School Sixth Form and studying A Level Further Maths

Circle areas

Can you work out the shaded area in the diagram (the line shown just touches the smaller circle)?



Find the value of

$$\frac{99}{100} \times \frac{80}{81} \times \frac{63}{64} \times \frac{48}{49} \times \frac{35}{36} \times \frac{24}{25} \times \frac{15}{16} \times \frac{8}{9} \times \frac{3}{4}.$$

Write your answer in the form $\frac{a}{b}$, where a and b are positive integers with no common factors other than 1.

A point E lies outside the rectangle $ABCD$ such that CBE is an equilateral triangle. The area of the pentagon $ABECD$ is five times the area of the triangle CBE.

What is the ratio of the lengths $AB : AD$?

Write your answer in the form $a : 1$.

A sequence is defined as follows:

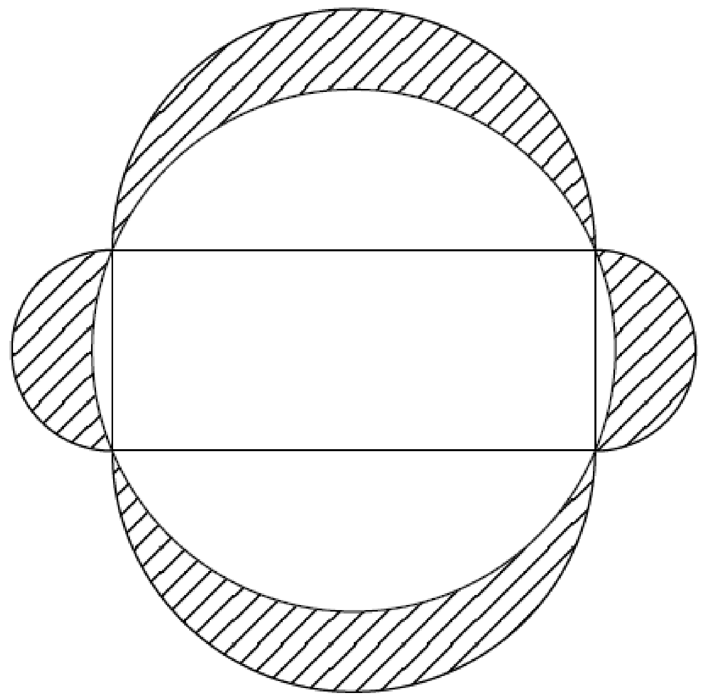
$u_1 = 123.$

For $n \geq 1$, define u_{n+1} = the sum of the squares of the digits of u_n.

For example, $u_2 = 1^2 + 2^2 + 3^2 = 14$, $u_3 = 1^2 + 4^2 = 17$.

What is the value of u_{100}?

Four semicircles are drawn on the sides of a rectangle with width 10 cm and length 24 cm. A circle is drawn that passes through the four vertices of the rectangle.



What is the value, in cm^2, of the shaded area?

Alfred, Brenda, Colin, David and Erica have to sit on a row of five chairs. Alfred does not want to sit next to Brenda. David does not want to sit next to Erica.

In how many ways can these five people arrange themselves and ensure the above conditions are met?

(a) Which positive integer in the range from 1 to 250 has more different prime divisors than any other integer in this range?

[3 marks]

(b) When $n = 5$ the product $n(n+1)(n+2)$ can be written as the product of four distinct primes. Indeed, when $n = 5$

$$n(n+1)(n+2) = 5 \times 6 \times 7 = 2 \times 3 \times 5 \times 7.$$

What is the least positive integer n such that $n(n+1)(n+2)$ can be written as a product of *five* distinct primes? [3 marks]

Find the value of

$$\left(\left(2^{\frac{3}{4}}+1\right)^2+\left(2^{\frac{3}{4}}-1\right)^2\right)\left(\left(2^{\frac{3}{4}}+1\right)^2+\left(2^{\frac{3}{4}}-1\right)^2-2^2\right).$$

The points $A(1,2)$ and $B(-2,1)$ are two vertices of a rectangle $ABCD$. The diagonal CA produced passes through the point $(2,9)$. Calculate the coordinates of the vertices C and D.

The inequalities $x^2 + 3x + 2 > 0$ and $x^2 + x < 2$ are met by all x in the region:

(a) $x < -2$;

(b) $-1 < x < 1$;

(c) $x > -1$;

(d) $x > -2$.

Year 11 Further Mathematics – Transition Materials

The following five problems are designed to get you to think in a little bit more depth about some mathematical concepts that you have met before. These problems will take you a little bit longer than the short ones and will require a more detailed solution. In some cases, they are quite open-ended.

You can attempt these problems in an order of your choice. If you would like to discuss your progress or request a hint, please email me.

All these problems are freely available at https://undergroundmathematics.org/

Problem 1: A tangent is…

You will be familiar with what a tangent is from GCSE. Tangents are used a lot in A level Maths because they represent the *rate of change* of a function. This all comes under the umbrella of *calculus*, which you will begin to study in Year 12.

The following problem is designed to get you to think about what a tangent is and what they look like. By working through this problem, you will improve your understanding of tangents, which will aid your progress with studying calculus.

A tangent is ...

Problem

Here are some suggestions of how we could define a tangent to a curve at a point.

(1) "A tangent is a straight line which only meets the curve at that one point."

(2) "A tangent is a straight line which touches the curve at that point only."

(3) "A tangent is a straight line which meets the curve at that point, but the curve is all on one side of the line."

(4) "A tangent is a straight line which meets the curve at that point, but near that point, the curve is all on one side of the line."

Which, if any, of these proposed definitions works for a circle?

For each one, can you find or sketch an example to show that the proposed definition does not work for all curves?

Can you come up with a better definition?

Problem 2: Scary Sum

At GCSE, you will have learned how to manipulate surds. The following problem is designed to get you to think about how to manipulate an expression involving surds without using a calculator.

Scary sum

Problem

Evaluate the sum

$$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\cdots+\frac{1}{\sqrt{15}+\sqrt{16}}.$$

(You might want to use a calculator to get an estimate of the answer, but in order to get the exact answer you will have to do it by hand!)

Can you find a similar sum that evaluates to 5?

Can you find a similar sum that evaluates to a number that is not an integer?

Problem 3: Parabella

Coordinate geometry is a massive topic in A level Maths. The skills you have learned at GCSE will be developed further and applied to a variety of problems.

Parabella

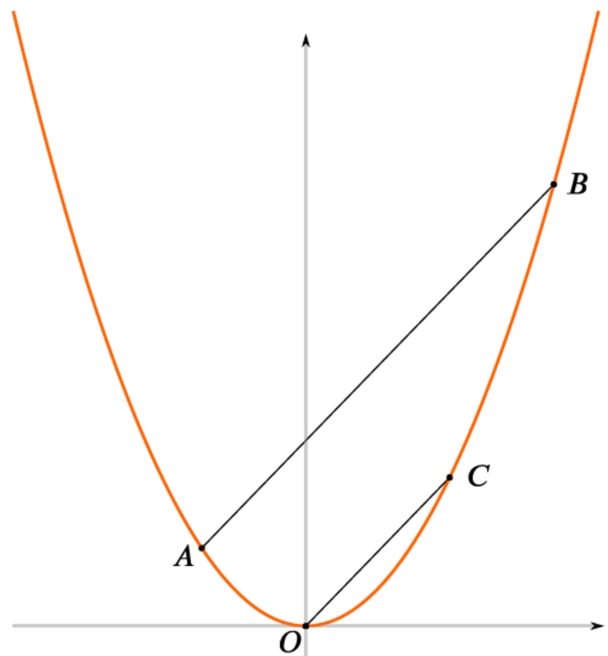
Problem

Take any two points A and B on the parabola $y = x^2$.

Draw the line OC through the origin, parallel to AB, cutting the parabola again at C.

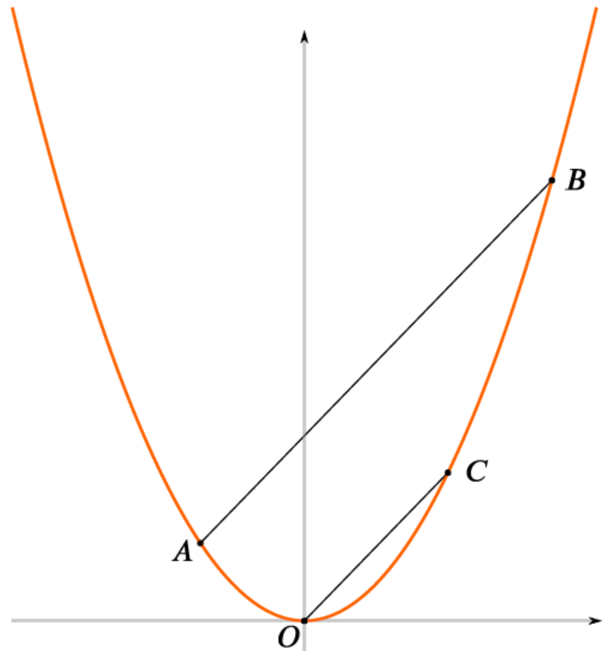
Let A have coordinates (a, a^2), let B have coordinates (b, b^2) and let C have coordinates (c, c^2).

Prove that $a + b = c$.



The parabella

Imagine drawing another parallel line DE, where D and E are two other points on the parabola. Extend the ideas of the previous result to prove that the midpoints of each of the three parallel lines lie on a straight line.



The parabella

Problem 4: Powerful Quadratics

You will be familiar with a number of ways of solving quadratic equations. This problem requires you to inspect an equation and to think about possible values that could satisfy it using your knowledge of indices.

There are more solutions than you might think…

Powerful quadratics

Problem

(i) Find all real solutions of the equation

$$(x^2 - 7x + 11)^{(x^2-11x+30)} = 1.$$

(ii) Find all real solutions of the equation

$$(2 - x^2)^{(x^2-3\sqrt{2}x+4)} = 1.$$

Problem 5: Two roots differ by 5

Solving equations is an important skill throughout mathematics, but in Further Maths, we begin to think in more depth about the relationship *between* the roots of an equation.

This problem is designed to get you to think about how quadratic and cubic equations are formed from their roots. This idea will be developed considerably in the first term of Further Maths as we consider *roots of polynomials* and begin to discover *complex numbers*.

(i) The equation $x^3 + ax + b = 0$ is satisfied by the values $x = 1$ and $x = 2$. Find the values of a and b; and find also the third value of x.

(ii) The equation $x^2 - 12x + k = 0$ is satisfied by two values of x that differ by 5. Find these two values of x, and the value of k.