HAMPSTEAD SCHOOL
Learning together Achieving together

Y12 – Y13

Summer Bridging Tasks

2023

Further Maths

Name: ______________________________________

- You should spend some time during the summer holidays working on the activities in this booklet.
- You will be required to hand in this booklet in your first lesson at the start of Year 12 and the content will be used to form the basis of your first assessments.
- You should try your best and show commitment to your studies.

Further Maths - Year 12 into 13 Summer Bridging Work

Task 1:

Re-do the three topic assessments which you struggled with the most, on A4 square paper.

Mark all three topic assessments with a green pen.

Please make sure you title and date each sheet of paper, and make sure your work is presented neatly.

Task 2 – Mech minor:

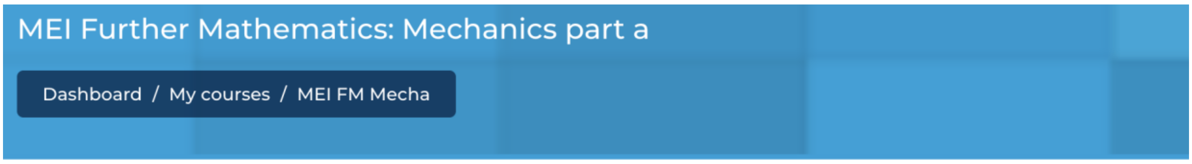
I found this knowledge organiser for Mech minor. Having said that, it needs to be adapted to suit our syllabus. Use your notes/Integral to adapt this knowledge organiser into one that can be used by us all in Year 13. I have uploaded a keywords glossary to help you.

Task 3 – Stats minor:

Use your notes/Integral to create a knowledge organiser for FM Stats minor. I have uploaded a keywords glossary to help you.

Task 4 – Applied revision questions:

Do the revision questions from Integral for Mech Minor AND Stats Minor.



Complete the questions on A4 square paper.

Please make sure you title and date each sheet of paper, and make sure your work is presented neatly.

You must bring these in along with your other Summer tasks in our first lesson back in September. After we have seen and marked your work, they will go into your independent learning folders.

Best of luck and enjoy your Summer break :)

Site: **Integral Mathematics Resources**
Course: **MEI Further Mathematics: Statistics part a (MEI FM Statsa)**
Glossary: **Glossary**

A

Alternative hypothesis

When you are carrying out a hypothesis test on a contingency table, the alternative hypothesis is that the two variables are not independent.

When you are carrying out a hypothesis test for goodness of fit, the alternative hypothesis is that the distribution specified in the null hypothesis is not an appropriate model for the data.

When you are carrying out a hypothesis test for correlation, with the null hypothesis $\mathrm{H}_0 : \rho = 0$, the alternative hypothesis could be $\mathrm{H}_1 : \rho < 0$ or $\mathrm{H}_1 : \rho > 0$ or $\mathrm{H}_1 : \rho \neq 0$.

B

Binomial distribution

The binomial distribution is an example of a discrete probability distribution. It may be used to model situations in which

- you are conducting trials on random samples of a particular size, n
- there are two possible outcomes, success (which has a fixed probability p) and failure (which has a fixed probability q, where $q = 1 - p$)
- the trials are independent of one another.

The discrete random variable X is the number of successes in the n trials.

The abbreviation $X \sim \mathrm{B}(n, p)$ means that X has a binomial distribution.

The probability that X takes the value r is given by

$$\mathrm{P}(X = r) = {}^n\mathrm{C}_r \, p^r q^{n-r}$$

For $X \sim \mathrm{B}(N, p)$, the expectation (mean) of X is given by $\mathrm{E}(X) = np$, and the variance is given by $\mathrm{Var}(X) = npq$.

Bivariate data

Data for which each item requires the values of two variables.

Bivariate Normal distribution

A bivariate data set comes from a bivariate Normal distribution if both variables are drawn from a Normal distribution. If this is the case, the scatter diagram for the data gives an approximately elliptical shape.

C

Chi-squared distribution

The χ^2 (chi-squared) distribution is formed when the squares of a set of independent standardised normal variables are added.

When carrying out a hypothesis test on a contingency table or a test for goodness of fit, the test statistic $X^2 = \Sigma \frac{(f_o - f_e)^2}{f_e}$ has an approximately χ^2 distribution, if the null hypothesis is true.

Contingency table

A two-way table when two variables are measured on a sample, each with two or more different categories of result. If there are m possible categories for the first variable, and n possible categories for the second variable, then a $m \times n$ contingency table can be drawn up, with each cell of the table containing an observed frequency: the number of instances in the sample for which that pair of values of the two variables occurs.

Controlled variable

A variable is controlled if it only assumes a set of predetermined values, such as the times at which measurements are taken.

Critical region

The range of values of the test statistic for which the null hypothesis is rejected.

Critical value

The critical value is the value of the test statistic for which you change from not rejecting the null hypothesis to rejecting it. This depends on the significance level chosen for the test.

In the case of a hypothesis test on a contingency table or for goodness of fit, the critical value is found from tables for the chi-squared distribution.

In the case of a hypothesis test for correlation, critical values for the product moment correlation coefficient and for Spearman's rank correlation coefficient can be found from tables.

D

Degrees of freedom

The number of independent variables involved.

For a $m \times n$ contingency table, the number of degrees of freedom is given by $(m-1)(n-1)$.

In a test for goodness of fit, the number of degrees of freedom is equal to the number of classes involved less the number of restrictions. There is always one restriction since the total of the frequencies is fixed, and if any parameters are estimated from the data, these result in further restrictions.

So the degrees of freedom = number of classes - number of estimated parameters - 1.

Dependent variable

In bivariate data, one variable (the dependent variable) may be dependent on the other (the independent variable). If this is the case, the dependent variable is usually plotted on the vertical axis.

Discrete random variable

A discrete random variable is a random variable which has a number of possible values, which can be listed (although the list could be infinite). Each value is associated with a probability. These probabilities must add up to 1.

A random variable is usually denoted by a capital letter, such as X or Y. The possible values taken by the random variable are sometimes written as $x_1, x_2, x_3, \ldots,$ The probability of a particular outcome x_i is written as $\mathrm{P}(X = x_i)$ or sometimes as p_i.

Discrete uniform distribution

For the discrete uniform distribution, there are a number of equally likely outcomes. Examples are the outcomes from using a fair dice or spinner.

For a discrete random variable X with a uniform distribution on $\{1, 2, \ldots, n\}$,

$$\mathrm{P}(X = r) = \frac{1}{n}$$

E

Expected frequency

When carrying out a chi-squared test on a contingency table or for goodness of fit, the expected frequencies f_e are the calculated frequency for each cell of the contingency table, or for each class of the distribution specified in the null hypothesis.

Extrapolation

Extrapolation is the extending of a relationship which has been established over a particular range of values, into a wider range. It is not usually appropriate to use extrapolated values for prediction purposes, as there may not be any evidence that the model continues to be a good one.

G

Geometric distribution

The geometric distribution may be used to model situations in which

- you are conducting trials for which there are two possible outcomes, success (which has a fixed probability p) and failure (which has a fixed probability q, where $q = 1 - p$)
- trials continue until the first success
- the trials are independent of one another.

The discrete random variable X is the number of trials up to and including the first success.

The abbreviation $X \sim \mathrm{Geo}(p)$ means that X has a geometric distribution.

The probability that X takes the value r is given by

$$\mathrm{P}(X = r) = pq^{r-1}$$

Goodness of fit

The chi-squared (χ^2) test for goodness of fit is a hypothesis test which determines whether or not a set of data fits a particular distribution, such as the binomial distribution, the Poisson distribution or the discrete uniform distribution.

H

Hypothesis test

A procedure in which you use statistical techniques to test a particular claim or theory (hypothesis).

I

Independent variable

In bivariate data, one variable (the dependent variable) may be dependent on the other (the independent variable). If this is the case, the independent variable is usually plotted on the horizontal axis.

L

Linear combination of random variables

The expectations of linear combinations of two random variables are given by

$$\mathrm{E}(aX + bY) = a\mathrm{E}(X) + b\mathrm{E}(Y)$$

$$\mathrm{E}(aX - bY) = a\mathrm{E}(X) - b\mathrm{E}(Y)$$

This result can be extended to any number of independent random variables.

The variances of the sums or differences of two independent random variables are given by

$$\mathrm{Var}(aX + bY) = a^2\mathrm{Var}(X) + b^2\mathrm{Var}(Y)$$

$$\mathrm{Var}(aX - bY) = a^2\mathrm{Var}(X) + b^2\mathrm{Var}(Y)$$

This result can be extended to any number of independent random variables.

Linear correlation

If a set of bivariate data plotted on a scatter diagram fall close to a straight line, then there is linear correlation. If the line has positive gradient, then the correlation is positive; if the line has negative gradient, then the correlation is negative.

If every point falls on a straight line, then there is perfect linear correlation, and the value of the product moment correlation coefficient is 1 (for positive correlation) or -1 (for negative correlation).

Linear function of a random variable

A linear function of a random variable X is of the form $aX + b$.

The expectation and variance of a linear function of X are given by:

- $\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$
- $\mathrm{Var}(aX + b) = a^2\mathrm{Var}(X)$

M

Mean of a discrete random variable

The mean (or expectation) of a discrete random variable is a measure of central tendency (average) for that random variable.

The expectation of a discrete random variable X is written as $\mathrm{E}(X)$ and is defined as

$\mathrm{E}(X) = \sum r\,\mathrm{P}(X = r)$ or $\mathrm{E}(X) = \sum x_i p_i$

Mean of the binomial distribution

For a random variable X which has a binomial distribution $X \sim \mathrm{B}(n, p)$, the mean (expectation) of X is given by

$$\mathrm{E}(X) = np$$

Mean of the discrete uniform distribution

For a random variable X with a discrete uniform distribution on $\{1, 2, \ldots, n\}$, the mean (expectation) of X is given by

$$\mathrm{E}(X) = \frac{n+1}{2}$$

Note that this is clear from symmetry.

Mean of the geometric distribution

For a random variable X which has a geometric distribution $X \sim \mathrm{Geo}(p)$, the mean (expectation) of X is given by

$$\mathrm{E}(X) = \frac{1}{p}$$

Mean of the Poisson distribution

For a random variable X which has Poisson distribution with parameter λ, the mean (expectation) of X is λ.

N

Non-linear correlation

There can be an association between variables which is not linear. In such cases the use of the product moment correlation coefficient is not appropriate.

In some cases of non-linear association, Spearman's rank correlation coefficient may be an appropriate measure of the correlation.

Null hypothesis

When you are carrying out a hypothesis test on a contingency table, the null hypothesis is that the two variables are independent.

When you are carrying out a hypothesis test for goodness of fit, the null hypothesis is that the data fit a particular distribution.

When you are carrying out a hypothesis test for correlation, the null hypothesis is $\mathrm{H}_0 : \rho = 0$, where ρ is the population correlation coefficient.

O

Observed frequency

When carrying out a chi-squared test on a contingency table or for goodness of fit, the observed frequencies f_o are the actual frequencies that occurred for each cell in the contingency table or each class in the experiment.

One-tailed test

A hypothesis test which looks for a change in a parameter in a particular direction.

In a one-tailed hypothesis test for correlation, the alternative hypothesis has the form $\mathrm{H}_1 : \rho < 0$ or $\mathrm{H}_1 : \rho > 0$, where ρ is the population correlation coefficient.

P

Parent population

The complete population from which a sample is taken.

Poisson distribution

A discrete random variable may be modelled by the Poisson distribution if events occur at random and independently of each other, in a given interval of time or space, and the average number of events in the given interval (denoted by λ) is uniform and finite.

If the random variable X is modelled by a Poisson distribution with parameter λ, then

$$\mathrm{P}(X = r) = \frac{\mathrm{e}^{-\lambda}\lambda^r}{r!} \quad r = 0, 1, 2, \ldots$$

The mean of the Poisson distribution and the variance of the Poisson distribution are both given by λ. This means that you can test the suitability of a Poisson distribution to model a set of data by finding out whether the mean and variance are close in value.

Population correlation coefficient

The correlation coefficient for a whole population, denoted by ρ.

(The value of r, the sample correlation coefficient, is used as a test statistic for the population correlation coefficient).

Probability distribution

The probability distribution of a random variable gives the probability of each of the possible outcomes. It may be given as an algebraic formula, or as a table of values.

Product moment correlation coefficient

A measure of the degree of linear correlation of a bivariate data set, denoted by r.

It is given by $r = \dfrac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$

where $S_{xx} = \Sigma x^2 - n\bar{x}^2$, $S_{yy} = \Sigma y^2 - n\bar{y}^2$, $S_{xy} = \Sigma xy - n\bar{x}\bar{y}$.

The value of r lies between -1 and 1, with 1 representing perfect positive linear correlation (all data falls on a straight line with positive gradient), 0 representing no correlation, and -1 representing perfect negative linear correlation (all data falls on a straight line with negative gradient).

R

Regression line

The least squares regression line is the equation of a line of best fit for a set of data. It is the line for which the sum of the squares of the residuals is as small as possible.

The equation of the least squares regression line is given by $y = \bar{y} + \frac{S_{xy}}{S_{xx}}(x - \bar{x})$

where $S_{xx} = \Sigma x^2 - n\bar{x}^2$ and $S_{xy} = \Sigma xy - n\bar{x}\bar{y}$.

Residual

The residual for a particular point on a scatter diagram is the vertical distance (positive or negative) from the point to a line, so that it is a measure of by how much the line misses the point. The sum of the residuals is zero.

The least squares regression line is the line for which the sum of the squares of the residuals is as small as possible.

S

Scatter diagram

A diagram used to represent bivariate data, in which the axes represent the two variables involved and each item of data is plotted using its two values as coordinates.

Significance level

For a hypothesis test, a significance level of $P\%$ means that the null hypothesis is rejected if the probability of a result at least as extreme as the result of the test is less than $P\%$.

Spearman's rank correlation coefficient

A correlation coefficient, denoted t_s, which is used for ranked data. It may also be used in situations when it is not appropriate to use the product moment correlation coefficient, such as if the data is not drawn from a bivariate Normal distribution.

Spearman's rank correlation coefficient is given by $r_s = 1 - \frac{6\Sigma d_i^2}{n(n^2 - 1)}$

where d_i is the difference in the ranks given to the i^{th} item.

Sum of Poisson distributions

If $X \sim \text{Poisson}(\lambda)$, and $Y \sim \text{Poisson}(\mu)$, and X and Y are independent of one another, then $X + Y \sim \text{Poisson}(\lambda + \mu)$.

This can be extended to any number of Poisson distributions, provided that they are independent of one another.

T

Test statistic

The test statistic for a hypothesis test on a contingency table or for goodness of fit is $X^2 = \Sigma\frac{(f_o - f_e)^2}{f_e}$, where f_0 is the observed frequency and f_e is the expected frequency for a particular cell or class.

The test statistic for a hypothesis test for correlation is the sample correlation coefficient (this could be the product moment correlation coefficient or Spearman's rank correlation coefficient).

Two-tailed test

A hypothesis test which looks for a change in a parameter without specifying a particular direction.

In a two-tailed hypothesis test for correlation, the alternative hypothesis has the form $\mathrm{H}_1 : \rho \neq 0$, where ρ is the population correlation coefficient.

V

Variance of a discrete random variable

The variance of a discrete random variable is a measure of spread for that random variable.

The variance of a discrete random variable X is written as $\text{Var}(X)$ and is defined as

$$\text{Var}(X) = \text{E}(X - \mu)^{2} \quad \text{or} \quad \text{Var}(X) = \text{E}(X^{2}) - \mu^{2}$$

where $\mu = \text{E}(X)$, the expectation of a discrete random variable for X.

Variance of the binomial distribution

For a random variable with a binomial distribution $X \sim \text{B}(n, p)$, the variance of X is given by

$$\text{Var}(X) = np(1 - p)$$

Variance of the discrete uniform distribution

For a random variable X with a discrete uniform distribution on {$1, 2, ..., n$}, the variance of X is given by

$$\text{Var}(X)=\dfrac{n^{2}-1}{12}$$

Variance of the geometric distribution

For a random variable with a geometric distribution $X \sim \text{Geo}(p)$, the variance of X is given by

$$\text{Var}(X)=\dfrac{1-p}{p^{2}}$$

Variance of the Poisson distribution

For a discrete random variable with a Poisson distribution with parameter λ, the variance is λ.

Site: **Integral Mathematics Resources**
Course: **MEI Further Mathematics: Mechanics part a (MEI FM Mecha)**
Glossary: **Glossary**

C

Centre of mass

The single point at which the whole weight of a body may be considered to act. In the case of a rigid body, this point is the balance point of the body.

Coefficient of friction

A constant, usually denoted by μ, which indicates the degree of roughness between two surfaces. The rougher the surfaces, the larger the value of μ.

Coefficient of restitution

A constant between 0 and 1, denoted by e. The coefficient of restitution between two surfaces (such as two balls, or a ball and a flat surface) is a measure of how bouncy the objects are when they collide.

If $e = 0$ the collision is perfectly inelastic and there is no bounce at all.

If $e = 1$ the collision is perfectly elastic and there is the maximum possible bounce with no change of kinetic energy.

Conservation of energy

The law of conservation of energy states that in the absence of any external forces other than gravity, total mechanical energy of a system is conserved.

Conservation of momentum

The law of conservation of momentum states that when there are no external influences on a system, the total momentum of the system remains constant.

Conservative force

A force which conserves mechanical energy, such as the force of gravity.

Coulomb's model for friction

The frictional force F between two surfaces depends on the coefficient of friction μ and the reaction force R between the surfaces.

$$F \le \mu R$$

Couple

Two forces with equal magnitude but acting in opposite directions. The resultant force is zero, but the two forces produce a turning effect.

D

Dimension

Dimensions are fundamental quantities, usually mass (M), length (L) and time (T), that are independent and that can be used to describe mechanical and other quantities.

Examples are:

- [volume] = L^3
- [momentum] = MLT^{-1}

The brackets mean "the dimensions of". Dimensions are closely related to the units used. For example, 1 m is a measure of length, and the SI unit for volume is m^3.

Dimensional consistency

Any equation or formula is dimensionally consistent when all its terms have the same dimensions. This is a requirement for any valid equation or formula.

Dimensionless

Angles and trigonometric functions are dimensionless because they are defined by dividing two quantities with the same dimensions (i.e. L divided by L). Numbers are dimensionless (including e and π).

Dissipative force

A force such as friction or air resistance which results in energy being dissipated as heat or sound.

Driving force

A driving force is an external force which causes an object to move - the force from a railway locomotive to pull a train, or from the engine of a car, are examples of driving forces.

F

Frictional force

Friction is a force that opposes surfaces sliding over one another. It happens because surfaces cannot be perfectly smooth, so one surface always "catches" a little on the other. If there were no friction, most machines would not work (and we wouldn't be able to walk along). For example, a car's wheels would just spin round and round, without moving the car, if there were no friction between the car's tyres and the road. When a car gets wheel-spin in slippery conditions it is because there is insufficient friction to propel the car forward, so the driving force from the engine is wasted spinning the wheels.

Friction is a special type of resistance force. For a stationary object, any frictional force is always at exactly the correct size and direction to keep the object stationary. However, the frictional force has a maximum value which depends on the value of the coefficient of friction. When the resultant force on an object exceeds this maximum, the object will move. The model of friction used at A level assumes that while an object is moving, the frictional force is constant at this maximum value.

Fulcrum

The pivot point of an object such as a lever or see-saw. If a force is applied which produces a moment, the object turns about the fulcrum.

G

Gravitational potential energy

The energy possessed by a body because of its position.

The gravitational potential energy of an object of mass m kg at height h m above a fixed reference level is given by mgh.

Graviational potential energy is measured in joules (J).

I

Impulse

The impulse due to a force F acting for t seconds is denoted by Ft.

The impulse on an object is also equal to the change in momentum of the object caused by that impulse.

Impulse is a vector quantity measured in Ns.

K

Kinetic energy

The energy possessed by a body because of its speed.

The kinetic energy of a body with mass m kg moving at speed v ms^{-1} is given by $\frac{1}{2}mv^2$.

Kinetic energy is measured in joules (J).

L

Lamina

A plane shape which may be considered to have negligible thickness, so that it need only be considered in two dimensions.

Limiting friction

The maximum possible value of the frictional force for two surfaces in contact. In the case where friction is limiting, the frictional force F is equal to μR, where μ is the coefficient of friction, and sliding either is occurring or is just about to occur.

M

Mechanical energy

Kinetic energy and gravitational potential energy are both forms of mechanical energy. When gravity is the only force acting on a body, total mechanical energy is conserved.

Method of dimensions

This is the method by which the proposed form of a formula which involves a product of powers of certain quantities can be found using considerations of dimensional consistency.

Model

A mathematical model is a way of describing the real world in terms of mathematics. All of the work you do in Mechanics involves the use of mathematical models.

In order to be able to make calculations about how objects in the real world behave, certain factors may be disregarded to simplify the situation, provided that they do not have a significant effect on the situation.

Modelling assumption

Examples of modelling assumptions are "strings connecting objects are inextensible and have no mass" or "air resistance can be ignored". These assumptions are made to simplify a situation to enable a mathematical model to be produced.

Moment

The moment of a force is the turning effect of the force about a fixed point. The moment is found by multiplying the magnitude of the force by its perpendicular distance from the fixed point.

The SI unit for a moment is the Newton metre (Nm). Anticlockwise moments are considered to be positive, clockwise moments negative.

Momentum

The momentum of a moving object is defined as mass × velocity.

Momentum is a vector quantity measured in Ns.

N

Newton's experimental law

A law connecting the relative speeds of two objects before and after a collision:

> speed of separation = e × speed of approach

where e is the coefficient of restitution.

P

Power

Power is the rate of doing work.

The power of a vehicle moving at speed v under a driving force F is given by Fv.

Power is measured in watts (W).

R

Rigid body

An object which is recognised as having size and shape, i.e. it cannot be considered as a particle. It is assumed not to be deformed when forces act on it.

A rigid body is in equilibrium if and only if there is no resultant force acting on it, and the total moment of the forces acting on it is zero at any point.

S

Speed of approach

The relative speed of two objects before a collision.

Speed of separation

The relative speed of two objects after a collision.

W

Work

The work done by a constant force F is given by

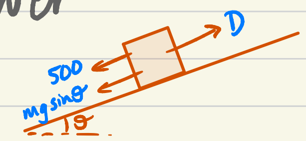
Work done = F × distance moved in the direction of the force.

Work is measured in joules (J).

Work-energy principle

The total work done by the forces acting on a body is equal to the increase in the kinetic energy of the body.

Power



Form 2 equations, then solve simultaneously

I) $P = Fv$ where F is the driving force, D

II) a) Constant speed
- Equilibrium, forces balanced

or b) Acceleration,
- $F = ma$, where F is the resultant

Elastics

Form 2 equations, then solve simultaneously

I) $T = \frac{\lambda x}{l}$

II) a) At rest
- Equilibrium, forces balanced

or b) Accelerating
- $F = ma$, where F is the resultant

Tips: Max velocity when $a = 0$, i.e. at its equilibrium position

Max acceleration when $v = 0$, i.e. just after it is released.

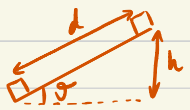
Energy

Types of Energy

KE $\frac{1}{2}mv^2$

GPE mgh or $mgd\sin\theta$

EPE $\frac{\lambda x^2}{2l}$



Work Done...

... against friction $F_r \times d$

... by engine/person $F \times d$

Work-Energy Principle

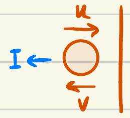
initial KE + initial GPE + initial EPE + work done by engine/person = final KE + final GPE + final EPE + work done against friction

$v=$ | $h=$ | $x=$ | $F=$, $d=$ | $v=$ | $h=$ | $x=$ | $F_r=$, $d=$

1D Collisions

Impulse-Momentum Principle

$I = m(v-u)$ $\quad I = Ft$



Take direction of impulse as positive direction

Principle of Conservation of Linear Momentum, PCLM

Initial momentum = final momentum Direction matters!

Newton's Law of Restitution, NLR

e is the coefficient of restitution

$0 \le e \le 1$ If $e = 0$, inelastic (not bouncy)

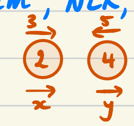
If $e = 1$, perfectly elastic (bouncy)

$e = \frac{\text{speed of separation}}{\text{speed of approach}}$

Strategy:

PCLM, NLR, simultaneous equations

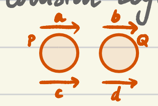
eg.



PCLM $3 \times 2 - 5 \times 4 = 2x + 4y$

NLR $e = \frac{y-x}{3+5}$

Collision Logic



$a > b$

$d > c$

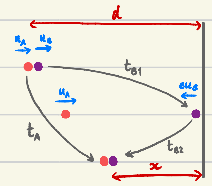
If P reverses $c < 0$

If Q stays same $d > 0$

Distance Problems

Constant velocity, use dist = speed × time

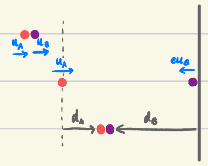
Collision, wall bounce, 2nd collision – "Find x"



Strategy 1 (Times)

- Compare the time it takes between collisions for A and B

$t_A = t_{B1} + t_{B2}$ Use $t = \frac{\text{dist}}{\text{speed}}$



Strategy 2 (Distance Ratios)

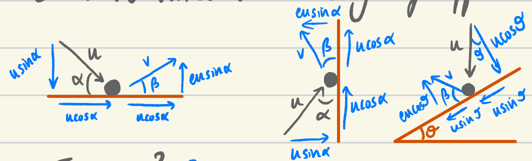
- If we know how far apart A and B are when B hits the wall, then speeds will be in same ratio as distances travelled i.e. $u_A : eu_B$ is equivalent to $d_A : d_B$

2D Collisions – one sphere

Impulse-Momentum Principle

$\underline{I} = m(\underline{v} - \underline{u})$

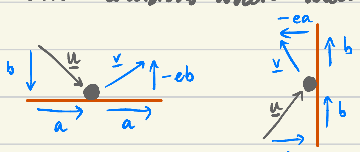
Collisions with known angle of approach



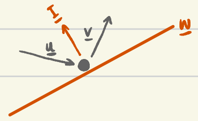
Find v? Pythagoras

β? $\tan^{-1}$

Vector collisions where wall is in $\underline{i}$ or $\underline{j}$ direction



Vector collisions where wall is not $\underline{i}$ or $\underline{j}$ direction



Conservation of velocity parallel to wall

$\underline{u}.\underline{w} = \underline{v}.\underline{w}$

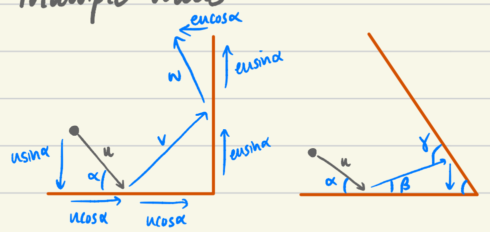
Impact law for perpendicular to wall

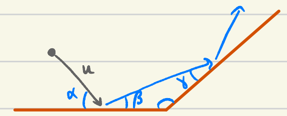
$-e\,\underline{u}.\underline{I} = \underline{v}.\underline{I}$

$\underline{I}$? Use $\underline{I} = m(\underline{v} - \underline{u})$

$\underline{I} \leftrightarrow \underline{w}$? Switch $\underline{i}$ and $\underline{j}$ comp and negate one.

Multiple Walls

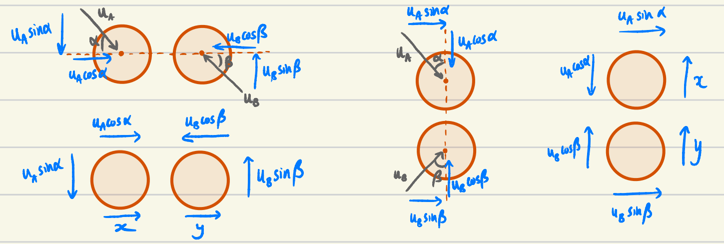




Use sum of angles in a triangle

2D Collisions – two spheres

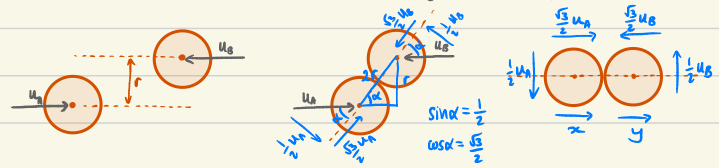
Collisions with known angles to line of centres



Strategy: PCLM, NLR, Sim equations

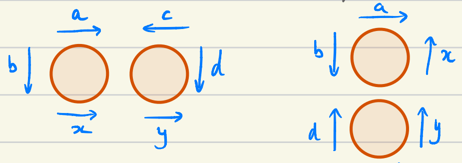
Put "back together" using Pythag. and Trig.

Collisions with unknown angles: geometric problems



... then same strategy as before

Vector collisions where line of centres is along $\underline{i}$ or $\underline{j}$



Vector collisions with unknown line of centres

3 of 4 known velocities? Vector version of PCLM

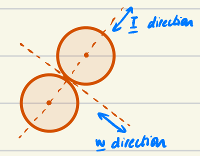
Direction of line of centres is same as direction of impulse, $\underline{I}$

Conservation perp. to line of centres

$(\underline{u}_A - \underline{u}_B).\underline{w} = (\underline{v}_A - \underline{v}_B).\underline{w}$

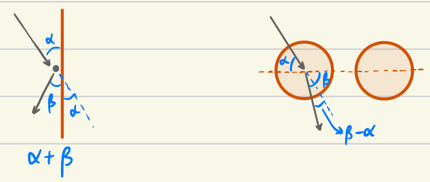
Impact law parallel to line of centres

$-e(\underline{u}_A - \underline{u}_B).\underline{I} = (\underline{v}_A - \underline{v}_B).\underline{I}$



Angles of Deflection

How much the original path is rotated/turned



If $\underline{u}$ and $\underline{v}$ are known...

$\cos\theta = \frac{\underline{u}.\underline{v}}{|\underline{u}||\underline{v}|}$